ANNUAL REPORT

OF THE

DEPARTMENT OF THE INTERIOR

FOR THE YEAR

1891

PRINTED BY ORDER OF PARLIAMENT.

OTTAWA:
PRINTED BY S. E. DAWSON, PRINTER TO THE QUEEN'S MOST EXCELLENT MAJESTY
1892.

[No. 13—1891.] *Price* 20 *cents.*

TABLE OF CONTENTS.

ber, 1891.

October, 1891.	Totals.
279	3,409
62	1,058
24	353
7	55
....	11
1	22
5	36
7	30
1	20
2	29
....	15
....	19
....	12
....	5
....	2
388	5,076

PART VI

DEPARTMENT OF THE INTERIOR

TOPOGRAPHICAL SURVEYS BRANCH

GENERAL REPORT OF OPERATIONS

FROM

1869 to 1889

Together with an Exposition of the System of Survey of Dominion Lands, and a Schedule of

DOMINION LAND AND TOPOGRAPHICAL SURVEYS

BY

W. F. KING, B.A., D.T.S., Chief Astronomer of the Department of the Interior,

AND

J. S. DENNIS, D.T.S., Chief Inspector of Surveys.

FEBRUARY, 1892.

DEPARTMENT OF THE INTERIOR,
TOPOGRAPHICAL SURVEYS BRANCH,
OTTAWA, 12th February, 1892.

SIR,—The surveys performed under the Topographical Surveys Branch of the Department of the Interior have been of varied character. The primary object has been the division of land for the purpose of settlement, and hence the bulk of the work has consisted of block, township outline, and township subdivision surveys, together with surveys of parishes and town plots. Other surveys have from time to time, in connection with these, become necessary, such as trail surveys, surveys of Indian reserves, exploratory and micrometer surveys, triangulation and photo-topographical surveys, determinations of latitudes and longitudes, &c.

Altogether a very large amount of work has been performed, and although much of the information concerning it is contained in the reports of the Department of the Interior published yearly, yet the necessity of a general report of all the operations, for use as a book of reference, has become apparent.

With the hope of supplying this want, we have the honour to submit herewith the first two sections of such report. The first section contains a historical narrative of the surveys, with schedules showing the names of all surveyors employed, together with the work performed by each, also all trail surveys, reserves for the Hudson's Bay Company, Indian reserves surveyed under the Department of the Interior, micrometer and exploratory surveys, correction of resurveys, acreage of the yearly surveys, and all surveys of parishes, town plots and other miscellaneous work. It also contains a list of all Dominion land and topographical surveyors.

The second section treats of the theory of the Dominion lands system of survey, and contains a number of geodetic tables useful in calculations connected with surveys under the system, and an explanation of the method of using them.

We have prepared this report in the hope that it will be found useful as a book of reference in all matters connected with the surveys, and to that end we have made it as complete as possible in the lines indicated.

It is proposed to supplement the information herein contained by the issue from time to time of additional sections giving fuller details of the various operations.

We have the honour to be, Sir,
Your obedient servants,

W. F. KING,
Chief Astronomer.
J. S. DENNIS,
Chief Inspector of Surveys.

E. DEVILLE, Esq.,
Surveyor General,
Topographical Surveys Branch.

CONTENTS.

SECTION I.

APPENDIX.

SECTION II.

CHAPTER I.

General Description of the System of Survey.

CHAPTER II.

Construction and Use of the Tables.

CHAPTER III.

Problems connected with the System of Survey.

Chapter IV.

Solutions of some Problems in Practical Geodesy.

Appendix.

SECTION I.

A SHORT HISTORY OF THE SURVEYS PERFORMED

UNDER THE

DOMINION LANDS SYSTEM

1869 TO 1889

BY

J. S. DENNIS, D.T.S., Chief Inspector of Surveys.

	Date of Commission.
...	April 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	do 14, 1872
...	Nov. 21, 1882
...	April 14, 1872
.	do 14, 1872
...	Feb. 21, 1888
...	April 14, 1872
...	Aug. 19, 1890
..	May 18, 1881
...	April 14, 1872
...	do 14, 1872
...	June 11, 1878
...	April 14, 1872
..	do 14, 1872
...	do 14, 1872
...	do 14, 1872
. .	Nov. 14, 1885
...	April 14, 1872
...	do 14, 1872
..	[illegible]. 22, 1882

SECTION II.

THEORY OF THE SYSTEM OF SURVEY

OF

DOMINION LANDS

WITH

GEODETIC TABLES AND NOTES ON THEIR USE

BY

W. F. KING, B.A., D.T.S.,
CHIEF ASTRONOMER OF THE DEPARTMENT OF THE INTERIOR.

SECTION II.

THEORY OF THE SYSTEM OF SURVEY OF DOMINION LANDS.

CHAPTER I.—GENERAL DESCRIPTION OF THE SYSTEM.

Size of the Township.

In the Dominion Lands surveys, the township contains thirty-six sections, each approximately one mile square, together with certain allowances for roads, and measures on each side six miles plus the road allowances.

Governing Lines—Initial Meridians and Base Lines.

The lines upon which the surveys are based are certain Principal or Initial Meridians which run from the International Boundary, or 49th parallel of latitude, northward indefinitely.

Along these meridians are placed the monuments marking the section and township corners in regular order northward from the boundary, from which also the townships are numbered.

There are also certain lines, called base lines, which run westward or eastward from the Initial Meridians, starting from them at distances apart of four townships; so that, the International Boundary Line being the first base line, the second base line lies between townships 4 and 5, the third between townships 8 and 9, and so on.

These base lines are surveyed as chords of the latitude circles which pass through their intersections with the Initial Meridian. The chords are one township (six miles together with the roads) in length, and hence an angle occurs on the base line at each township corner. Along the base lines, as on the Initial Meridians, the section and township corner monuments are placed at their regular distances.

Meridian Boundaries.

The eastern and western boundaries of townships are true meridians which start from the base line and are continued on each side thereof for two townships, when they encounter the meridians drawn in the same way from the next base line, but do not meet them exactly, since, on account of the convergence and divergence of meridians, the extremity of the line drawn south from the northerly base line passes to the west of that drawn north from the southerly base line.

Correction Lines.

Hence a "jog" occurs on that township line which lies midway between the base lines. This township line is called a correction line, for on it not only the jogs due to the system itself, but also all errors in survey, whether in the chainage or in the azimuth of the lines, are allowed to fall and are so prevented from accumulating to such an extent as to deform other townships except those on whose outlines they occur.

Northern and Southern Boundaries of Townships.

The northern and southern boundaries of townships are straight lines (or great circles of the sphere) joining the corresponding points on the east and west meridian boundaries.

Form and Dimensions of Townships.

Townships are therefore quadrilaterals, having their east and west sides true meridians, and in length equal to six "sections" (that is six miles together with the roads), and having their north and south sides inclined at equal angles to these meri-

dians, while the northern boundary is somewhat shorter than the southern boundary, these lengths varying from 480 chains plus the roads on the base line to about 180 links more on the next correction line south, and about 180 links less on the next one to the north. The angles of the township differ from 90° by about 4′ only.

These are the theoretical dimensions and form of the township. Of course, the lengths of the lines and the magnitude of the angles may differ from theory from the effect of errors in surveying, but the closings on correction lines cut out these errors and prevent them from so accumulating as to materially deform the townships.

Townships are designated by their numbers counting north from the 49th parallel with the number of the "Range" in which they lie, these ranges being counted east or west from the Initial Meridian.

Different Systems of Survey.

Since the surveys in Manitoba and the Western Territories of Canada were initiated in 1870, changes have been made from time to time in the system, as regards the number and width of the road allowances, as well as in the manner of surveying townships and sections. There have thus been three systems of survey, generally called the first, second and third systems from their order in time.

Distinctions between the Systems.

In the first and second systems the roads are one and a half chains wide, and are placed between all sections on both north and south, and east and west lines.

In the third system, which covers the entire area of Manitoba and the Western Territories, except the comparatively small area previously surveyed under the first and second systems, the roads are only one chain wide, and are placed along each alternate east and west section line, and along each north and south line.

So the townships of the first and second systems are 489 chains each way, while those of the third system are 483 chains from north to south, and 486 from east to west (these widths being, as above explained, subject to increase or decrease from divergence or convergence of meridians).

The second system differs from the first in the manner of subdividing the township into sections. In the first system, the interior lines forming the eastern boundaries of sections are drawn parallel to the eastern boundary of the township, so that all the deficiency or surplus caused by convergence of meridians, is left in the tier of quarter sections adjoining the western boundary of the township.

In the second system the eastern boundaries of sections are true meridians.

In the third system also the interior lines are true meridians.

In all three systems the northern and southern boundaries of sections are straight lines connecting points on the eastern or western boundaries, which have been established by chainage.

In all the systems the sections in a township are designated by numbers from 1 to 36, beginning with 1 at the south-east corner of the township and counting west and east alternately across the township to 36 in the north-east corner.

Position of Posts with regard to the Road Allowances.

The posts for section corners are placed on the south and west sides of the road allowance, each section post governing the corner of four sections, except on correction lines, where posts stand on the north side of the road to mark the boundaries of sections on the north side of the road. Also on the lines between different systems of survey, posts are placed on both sides of the road allowance.

But, in general, the post marks the south or west side of a road allowance, or in other words, stands at the north-east corner of a section. The quantities given in the appended tables always refer, unless otherwise stated, to the northern and eastern boundaries of sections or townships.

Fourth System of Survey.

There is a fourth system of survey, which is in force in the Canadian Pacific Railway belt in British Columbia. This system is exactly similar to the third system, as to the manner of surveying townships, and the townships are of the same dimensions; but the roads are thrown into the sections, so that every section measures 80·50 chains from north to south, and 81 from east to west, subject to deficiency or surplus from converging or diverging meridians.

Thus in the fourth system the quarter section and section posts on a base line, beginning at the easterly corner of a township and going west, stand at distances 40·50, 40·50, 40·50, 40·50, &c., while in the third system they stand at 40, 41, 40, 41, &c., the only difference being in the position of the quarter section posts. On the meridian outline of a township, in the fourth system, beginning at the southerly corner and going north, the posts stand at 40·25, 40·25, 40·25, 40·25, &c., while in the third system they stand at 41, 40, 40, 40, &c. Here there is a difference in the position of the quarter section corners, and each alternate section corner. The greatest difference in the position of any post is 75 links. The tables made for the third system, therefore, answer for the fourth also, except the tables of latitudes and longitudes, which will require correction in cases where the highest degree of accuracy is desired.

Fifth System of Survey.

This system is applied to the survey of certain townships in the lower valley of the Fraser River in British Columbia. There are no roads. Each section is 80 chains square, and the townships, of 36 sections each, are based upon the 49th parallel and an Initial Meridian called the Coast Meridian.

Advantages of the Dominion Lands System of Survey.

Some of the advantages of the Dominion Lands system of survey (especially the third system) are these:—

The boundaries of townships are straight lines (that is, great circles or surveyors' transit lines), and the interior lines also are straight for the greatest possible distance. The straightness of lines greatly facilitates the picking up of a line and its re-establishment when some of the posts have been removed or destroyed.

Directions of analogous lines in two townships or two sections are the same, or nearly so. This simplifies the original survey and facilitates resurvey. Lines are also referred to the astronomic meridian, thereby avoiding the confusions and errors arising in many of the older settled parts of the Dominion from the use of the variable magnetic meridian.

The parcels of land are, as nearly as possible, equal in area and similar in form, and permit of a simple system of numbering, by which descriptions are facilitated. The parcels of land are also square, or nearly so—the shape most suitable, on the whole, for farms.

The surveys of different townships and different parts of the country are independent, or nearly so. Errors are cut out, and not carried forward throughout the system, and the survey of an isolated tract may be made without waiting for the complete survey of all the country intervening between it and the initial point of the system, and without fear of a gore or overlap, when the intermediate district is surveyed.

Tables.

Another result from the similarity of townships to each other is the simplicity of the tables giving the azimuths and lengths of lines. Such tables are indispensable in surveys to be made on a very large scale and by a great number of surveyors.

Tables of azimuths and lengths of lines were calculated by the writer, and published as an appendix in the Annual Report of the Minister of Interior for 1879. These tables were calculated for the first and second systems of survey.

In 1881 the change in the system of survey necessitated a recalculation, so that the tables might serve for the third system of survey. The new tables were printed in the Manual of Surveys issued by direction of the Minister of Interior in 1881 (a second edition in 1883).

Since the tracts of country set aside for the first and second systems have not yet been completely surveyed, it has been deemed advisable to reprint here the tables for the first and second systems along with those of the third system for the sake of ready reference. The tables in the appended collection have been carefully checked. Table I, the general geodetic table, not referring to any particular system of survey, has been carefully recomputed, and has been extended so as to cover the whole of Canada from its most southerly point, Point Pelee, in Lake Erie, in latitude 42°, to latitude 70°.

Limits of the Different Systems of Survey.

The operation of the first system of survey is restricted to the area bounded as follows, viz.:—

To the south by the International Boundary Line; to the west by the Second Meridian as far as the eighth correction line; by said correction line as far as the meridian between Ranges 28 and 29 west of the Principal Meridian; by said meridian, between Ranges 28 and 29, as far as the seventh correction line; by said correction line as far as the meridian between Ranges 7 and 8, east of Principal Meridian; by said meridian, between Ranges 7 and 8, as far as the north boundary of Township 19; by the north boundary of Township 19, in Ranges 8, 9 and 10, east of the Principal Meridian as far as the meridian between Ranges 10 and 11, east of the Principal Meridian; by said meridian, between Ranges 10 and 11, as far as the third correction line; by said correction line as far as the eastern boundary of the Province of Manitoba; by said eastern boundary as far as the International Boundary Line.

Also Townships 44, R. 21; Tp. 45, R. 21, 22, 27, 28; Tps. 46 and 47, R. 25, 26, 27 and 28; Tp. 47, R. 24, and Tp. 48, R. 24, 25, 26 and 27, west of the Second Meridian.

Townships 42 to 47 inclusive, R. 1; and Tps. 43 and 44, R. 2 and 3, west of the Third Meridian.

The second system of survey is similar in all respects to the first system, except in regard to the deficiency or surplus from converging or diverging meridians which is distributed equally between all quarter sections as in the actual system.

The operation of the second system of survey is restricted to Tps. 1 and 2, R. 1 to 8 inclusive; Tps. 19 to 30, R. 1 to 12 inclusive; and Tps. 27 to 30, R. 13 to 16 inclusive; the above ranges being all west of the Second Meridian.

The fourth system includes the belt twenty miles on each side of the Canadian Pacific Railway, west of the summit of the Rocky Mountains.

The fifth system, as already stated, applies to a few townships only in south-western British Columbia.

The third system is applied to all Dominion lands not included in the first, second, fourth and fifth systems.

CHAPTER II.

CONSTRUCTION AND USE OF THE TABLES.

TABLE I.

Length of Arcs of Meridians, Parallel, &c., in Different Latitudes.

According to Col. A. R. Clarke, R.E., in his "Comparison of Standards of Length" (1866), the spheroid of revolution most nearly approaching the form of the earth has for its major or equatorial semi-axis 20926062 feet, and for its minor or polar semi-axis 20855121 feet.

Representing the major and minor axis by a and b respectively, we have for the compression, $C = \frac{a-b}{a} = \frac{1}{294 \cdot 98}$, and the eccentricity e is given by the formula

$$e^2 = \frac{a^2 - b^2}{a^2} = \frac{1}{148} \text{ nearly.}$$

The unit of measure in the Dominion Lands surveys is the Gunter's, or sixty-six feet chain. The equatorial semi-axis in chains is 317061·545 +

Representing by ϕ the geographical latitude of a place, or the angle which its vertical line makes with the plane of the equator, we have for the radius of curvature of the meridian

$$R = \frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{\frac{3}{2}}},$$

for the length of the normal to the meridian terminated by the minor axis

$$N = \frac{a}{(1-e^2 \sin^2 \phi)^{\frac{1}{2}}},$$

and for the radius of the parallel of latitude ϕ

$$P = N \cos \phi.$$

The length in chains of one second of latitude is equal to $R \sin 1''$; one second of the great circle perpendicular to the meridian is equal to $N \sin 1''$; and one second of longitude is equal to $P \sin 1''$. The logarithms of these quantities are placed in the second, third and fourth columns of Table I. They have been calculated by means of the logarithmic expansions of R and N.

Thus putting n for $\frac{a-b}{a+b}$ we have

$$\log (R \sin 1'') = \log a + \log \sin 1'' - M\left(n + \frac{3n^2}{2}\right)$$
$$= 3\,M\left(n \cos 2\phi - \frac{n^2}{2} \cos 4\phi\right) + \&c.$$

where M is the modulus of the common system of logarithms, and powers of n higher than the second are neglected as being insensible in the eighth decimal place.

Substituting the value of a in chains, as given above, and taking

$$n = \frac{a-b}{a-b} = \frac{1}{588 \cdot 96}, \text{ we get}$$

$$\log (R \sin 1'') = 0 \cdot 18597916 - 0 \cdot 00221218 \cos 2\phi + 0 \cdot 00000188 \cos 4\phi.$$

In calculating the two last terms by logarithms five places are sufficient.

For $N \sin 1''$ we have

$$\log (N \sin 1'') = \tfrac{1}{3} \log (R \sin 1'') + \tfrac{2}{3} \{\log a + \log \sin 1'' + 2\,Mn\}$$
$$= \tfrac{1}{3} \log (R \sin 1'') + 0 \cdot 12546215.$$

For P $\sin 1''$; $\log P \sin 1'' = \log (N \sin 1'') + \log \cos \phi$.

The calculation has been made to eight places of decimals to ensure accuracy in the seventh place. In tabulating, the eighth figure has been dropped.

The calculation of the logarithms of $R \sin 1''$ and $N \sin 1''$ has also been made directly from the formulæ for R and N, by the use of a subsidiary angle.

Thus, finding an angle ψ such that $\sin \psi = e \sin \phi$ we have

$$R \sin 1'' = a(1-e^2) \sec^3 \psi \sin 1''$$
$$N \sin 1'' = a \sec \psi \sin 1''.$$

Seven figure logarithms were used, and consequently the results could not be depended upon to the seventh figure, but they have been serviceable as a check upon the series computation.

Log $N \sin 1''$, log $P \sin 1''$ and log $R \sin 1''$ are given in the table for every 10′ of latitude from 42° to 70°. Their values for intermediate latitudes can be obtained by simple interpolation. Where, however, log $P \sin 1''$ is required with accuracy for an intermediate latitude, it is better first to obtain log $N \sin 1''$ for that latitude by interpolation from the table and then to add log cos ϕ.

Under the heading "Chains in 1″" are given the natural numbers corresponding to the logarithms of $R \sin 1''$ and $P \sin 1''$. These natural numbers are useful in reducing small differences of latitude and longitude to chains by simple multiplication, being preferable in many cases to the logarithms.

The converse operation of reducing short distances north and south or east and west to seconds of latitude or longitude may be performed by multiplying by the quantities in the two columns headed "seconds in one chain." These columns contain the reciprocals of the quantities in the columns "chains in one second."

In the last two columns of the table are given the lengths of one degree of latitude and longitude in English miles.

Radius of Curvature of a Section of the Spheroid inclined at any angle to a Meridian.

In some operations it is necessary to find the radius of curvature of the trace on the earth's surface of a "straight" or "transit" line making a given angle with the meridian.

Representing this radius of curvature by S, and θ being the angle with the meridian, we have the formula

$$\frac{1}{S} = \frac{\cos^2\theta}{R} + \frac{\sin^2\theta}{N}$$

and introducing an auxiliary angle X determined by the formula

$$\tan X = \sqrt{\frac{R \sin 1''}{N \sin 1''}} \tan \theta, \text{ we have}$$

$$S \sin 1'' = N \sin 1'' \frac{\sin^2 X}{\sin^2 \theta}$$

a formula adapted for ready calculation by means of logarithms.

Radius of Spherical Curvature.

The mean of the values of S when θ is given all possible values is $\sqrt{NR}$. This is the radius of curvature of the surface or the radius of the sphere to the surface at a given point. Its logarithm is readily found from Table I, being the arithmetical mean of the logarithms of N and R.

TABLE II.

Corrections to Table I for Change in Elements of Figure of Earth.

In Table I the data used are Clarke's 1866 values, viz.:—

$$a = 20926062 \text{ feet}$$

$$n = \frac{1}{588{\cdot}96}$$

and all the following tables are based on Table I, and therefore on these values. Clarke's later values (Geodesy, 1888) are,

$$a = 20926202 \text{ feet.}$$

$$n = \frac{1}{585{\cdot}93}$$

If, for any purpose, it is desired to use these values, Table I can be corrected by means of Table II, which has been computed thus:

Differentiating the formulæ,

$$\log R \sin 1'' = \log a + \log \sin 1'' - M(n + \tfrac{3}{2}n^2) - 3Mn \cos 2\Phi + \tfrac{3}{2}Mn^2 \cos 4\Phi$$

$$\log N \sin 1'' = \log a + \log \sin 1'' + M\left(n - \frac{n^2}{2}\right) - Mn \cos 2\Phi + \tfrac{1}{2}Mn^2 \cos 4\Phi$$

and putting $\frac{1}{n} = p$, we have

$$d(\log R \sin 1'') = M\frac{da}{a} + Mn^2\,dp + 3Mn^2 \cos 2\Phi dp$$

$$d(\log N \sin 1'') = M\frac{da}{a} - Mn^2\,dp + Mn^2 \cos 2\Phi dp$$

M being the modulus of the common system of logarithms. Terms involving the cubes and higher powers of n are insensible and may be neglected.

To change Clarke's earlier to his later values, we have

$$da = +140 \text{ (feet)}$$
$$dp = -3{\cdot}03$$
$$a = 20926062 \text{ (feet)}$$
$$n = \frac{1}{588{\cdot}96}$$
$$\text{and } M = 0{\cdot}43429448$$

whence $d \log (R \sin 1'') = -{\cdot}00000089 - {\cdot}00001138 \cos 2\Phi$

$d \log (N \sin 1'') = +{\cdot}00000670 - {\cdot}00000379 \cos 2\Phi$

These quantities are tabulated in Table II, with the proper signs of application to $\log R \sin 1''$ and $\log N \sin 1''$ in Table I.

TABLE III.

Latitudes of Base and Correction Lines and Lengths of Arcs of Meridian, Parallel, &c., for First and Second Systems of Survey.

This table is constructed for the first and second systems of survey only. It accordingly stops at the 13th Base, Township 48, north of which there are no surveys under these systems.

Each township measuring 489 chains each way, the 1st correction line is 978 chains north of the 49th parallel.

The latitude of the 1st correction line is therefore $49° + \dfrac{978}{R \sin 1''}$.

Here $R \sin 1''$ must be taken from Table I for the middle latitude between the 1st base and the 1st correction line. For accuracy it is therefore necessary to compute an approximate difference of latitude, using an approximate value of $R \sin 1''$. For instance $R \sin 1''$ may be taken from the table for latitude 49°.

The approximate difference of latitude being thus determined, the middle latitude is found from it (this being a sufficiently close approximation), and the final $R \sin 1''$ is taken from Table I for that latitude. Then dividing 978 by this we have a very close approximation to the difference of latitude between the base and the correction line.

From the latitude thus obtained of the 1st correction line, that of the 2nd base line is found by a similar process, and so on in succession as far as the table extends.

The table is checked by applying the same process to a longer distance than 978 chains. For example the latitude of the 6th base can be directly determined from that of the first by using 9,780 chains instead of 978. When long distances are thus taken, a second approximation to the middle latitude may become necessary.

The columns $\log N \sin 1''$ and $\log R \sin 1''$ are taken from Table I by interpolation, and $\log P \sin 1''$ is found by adding $\log \cos \Phi$ to $\log N \sin 1''$.

The width of a township along a base line is 489 chains. The longitude corresponding to this length measured along the parallel of latitude is given in the column headed "Longitude covered by 489 chains westing," not only for the base lines but also for the correction lines.

The longitude for 489 chains, along a base line, is the longitude covered by one range of townships. Along a correction line it does not correspond to the longitude covered by a range, since the width of a township along a correction line is greater or less than 489 chains according as the township north or south of the correction line is considered. The tabulated quantity however for correction lines can be used to calculate the narrowing or widening of sections at the correction lines.

The township width 489 chains is measured along the base line which has such azimuth that its terminal point falls in the same latitude as its initial point.

Thus every township corner along a base line has the same latitude, and the base line is a succession of chords of the latitude circle.

The difference of longitude between one township corner and the next is given by the formula

$$d\lambda = \frac{486}{P \sin 1''}$$

It is assumed here that the chord of the arc of the latitude circle is equal to the arc. That the difference between the chord and the arc is inappreciable may be shown thus:

By spherical trigonometry

$$\sin \frac{\text{chord}}{2N} = \sin \frac{d\lambda}{2} \cos \Phi$$

whence $$\text{chord} = N \cos \Phi d\lambda - N \cos \Phi \sin^2 \Phi \frac{d\lambda^3}{24}$$

$$= \text{arc} - \text{arc} \times \frac{d\lambda^2}{24} \sin^2 \Phi$$

So that the difference between the chord and the arc is equal to

$$\text{arc} \times \frac{d\lambda^2}{24} \sin^2 \Phi$$

$d\lambda$ being in a circular measure.

For a chord of 489 chains this amounts to less than one-hundredth of a link.

The chord always lies north of the arc. The distance between them is greatest at their middle points, amounting there to about 10 links. Hence, at the International boundary line, which is the first base line, since the actual territorial boundary is the curve, and the base line a series of chords, the road allowance which lies along the north side of this base is increased in width by 10 links at the middle of the chords.

The non-coincidence of the chord and arc also has the effect of increasing and decreasing the widths of roads on correction lines. This will be referred to again.

In the first column of Table III are given, for convenience, the numbers of the townships corresponding to the several base and correction lines. Thus the sixth base is the northern boundary of Township 20, and so on.

TABLE IV.

Latitudes of Base and Correction Lines, &c., for 3rd and 4th Systems of Survey.

This is exactly similar to Table III, except that it is made for the third system of survey, where the widths of townships are 486 instead of 489 chains, and their depths, in a north and south direction, 483 instead of 489 chains.

This table also applies, without change, to the fourth system (British Columbia).

In this table, as well as in Table III, the latitudes given are those of the line of posts on the south side of the road allowance. To get the latitude of the posts north

of the road on correction lines, the latitude of the correction line, as given in the table, must be corrected by adding the equivalent in latitude of the width of the road, *i.e.*, one chain and a-half for the first and second systems (Table III), and one chain for the third system (Table IV).

TABLE V.

Chord Azimuths, &c., for Base Lines, First and Second Systems of Survey.

The extremities of the township chord, as above stated, are in the same latitude. Hence the chord is equally inclined to the meridians passing through its terminal points, and its azimuth, east or west of north, is equal to the complement of half the change in azimuth, that is, of half the "convergence of meridians."

Let dA represent the change in azimuth or convergence of meridians, $d\lambda$ the difference of longitude, and Φ the latitude.

Then, by spherical trigonometry,

$$\tan \tfrac{1}{2} dA = \tan \tfrac{1}{2} d\lambda \sin \Phi,$$

whence, by expansion of the tangents in terms of the arcs,

$$dA = d\lambda \sin \Phi + \frac{d\lambda^3}{12} \sin \Phi \cos^2 \Phi$$

or, if dA and $d\lambda$ be expressed in seconds,

$$dA = d\lambda \sin \Phi + \frac{d\lambda^3}{12} \sin \Phi \cos^2 \Phi \sin^2 1''.$$

The second term is inappreciable, amounting in latitude 51° to less than one ten-thousandth of a second.

$$\therefore dA = d\lambda \sin \Phi.$$

The convergence or "deflection" (dA), given in Table V, is thus calculated from the difference of longitude ($d\lambda$) in Table III.

The "chord azimuth" is the complement of half the deflection.

The chord azimuth and the deflection are given in the table in degrees, minutes and seconds, as well as in decimals of a degree, for sexagesimally and decimally divided instruments respectively.

In the survey of a base line, the surveyor, when he arrives at a township corner, deflects his line to the north through an angle equal to the "deflection," and thus establishes in azimuth the chord across the next range of townships.

This deflection angle may be turned with the instrument, but more readily by the use of the "deflection offsets" in the table. The tabulated offset is the linear distance in inches between one of the chords and the prolongation of the other, at one chain from the township corner.

Their distance apart at any point is found by multiplying the tabulated offset by the distance, expressed in chains, of the point from the township corner.

For example, if the instrument is standing on the prolongation of the first chord at 5 chains past the corner, and the back picket be 15 chains on the other side of, that is, behind the corner, then the instrument must be moved north five times, and the back picket south fifteen times, the "deflection offset for one chain." The line of the instrument and picket is now in the correct bearing for the prolongation of the base line.

The angle is thus turned as accurately as a straight line can be produced with the instrument, and much more accurately than the angle can be measured with the graduated arc, while the setting of the instrument at the corner (which may be in low ground, unsuitable for accurate line production) is rendered unnecessary.

"Longitude covered by one range" in the seventh column is merely the longitude in the seventh column of Table III, reduced to time by dividing by 15. This gives the number of seconds which a watch will gain or lose on local time in being carried across a range. The gain or loss in travelling over any other distance along is proportional to the distance. The column is added for astronomical purposes, especially the determination of azimuth by observation of Polaris at any hour angle.

This Table V applies to the first and second systems of survey.

TABLE VI.

Chord Azimuths, &c., for Base Lines, 3rd and 4th Systems of Survey.

This table is exactly similar to Table V, but is made for the third system of survey.

The calculation is made by the same formulæ, changing only the width of the range, which is 486, instead of 489 chains, and using the latitudes of the base lines from Table IV, instead of those from Table III.

$$d\lambda = \frac{486}{P \sin 1''} \qquad dA = d\lambda \sin \Phi.$$

This table also applies to the fourth system.

TABLE VII.

Chord Azimuths, Jogs, &c., for Correction Lines, 1st and 2nd Systems of Survey.

This table gives quantities for correction lines similar to those given in Table III for base lines. This table applies to the first and second systems of survey.

The correction lines are posted on both sides of the road. The chord azimuths and deflections are given for the south side of the road, which is that side for which the latitudes of correction lines are given in Table III.

The calculation of the chord azimuth for correction lines is somewhat different from that for base lines.

For the base lines we have

$$d\lambda = \frac{489}{P \sin 1''}$$

$$\text{deflection} = d\lambda \sin \Phi.$$

For the correction lines, one range is not 489 chains, but the distance between meridians which include 489 chains on the nearest base line.

Hence in the formulæ—

$$d\lambda = \frac{489}{P \sin 1''}$$

and deflection $= d\lambda \sin \Phi = \frac{489}{P \sin 1''} \sin \Phi$, we must take $P \sin 1''$ for the next base line south of the correction line, if the difference of longitude and the deflection for the south side of the correction line road are required; while for the north side of that road we must take $P \sin 1''$ for the next base line north. Φ, of course, is the latitude of the correction line itself.

The length of one range on the correction line is $d\lambda \times P \sin 1''$

If, then, P_1 and P_2 represent the radius of parallel for the base lines next north and south, respectively, P that for the correction line itself

$$d\lambda_1 = \frac{489}{P_1 \sin 1''}$$

$$d\lambda_2 = \frac{489}{P_2 \sin 1''}$$

and we have for the length of one range on the correction line

$$\text{North side} = \frac{489}{P_1 \sin 1''} \times P \sin 1''$$

$$\text{South side} = \frac{489}{P_2 \sin 1''} \times P \sin 1''$$

The values of these quantities are tabulated in the seventh and eighth columns of Table VII.

For extreme accuracy $P \sin 1''$ for the north side of the road should be taken out for a latitude greater by 1·50 chains, or 0″·98 greater than that tabulated in Table III; but the difference in the result would be almost inappreciable.

The difference of length of the township lines north and south of the correction line road gives the overlap or jog.

The jog for one range is given in the ninth column of the table. As this jog occurs in each range of townships, its value at any range is the product of the jog for one range by the number of ranges.

The excess of the length of the north side over, or the defect of the south side from 489 chains, is the linear divergence or convergence of the township lines. Since there are twelve half sections in a township side, the convergence or divergence for one-half section is one-twelfth of the convergence or divergence for the township, or one-twenty-fourth of the jog, the excess of the north side and the defect of the south side being very nearly, though not quite, equal.

This convergence or divergence for one half section is entered in the tenth column of the table. It is used in the second system, where the surplus or deficiency caused by the convergence of meridians is divided equally among all the quarter-sections. Hence, in surveying a correction line under the second system, the width of each quarter section (exclusive of the roads) is forty chains *plus* or *minus* this tabulated quantity. The surplus or deficiency on the township line midway between the base and the correction line is half of that on the correction line.

In the first system the whole of the surplus or deficiency is thrown into the western tier of quarter sections. This surplus or deficiency is the difference between 489 chains and the quantities in the seventh and eighth columns of Table VII. For example, on the north side of the road on the 1st correction line the surplus is 1·75 chains, and the westerly quarter section of the township is therefore 41·75, all the others being 40 chains.

It is to be observed that in all cases the whole divergence or convergence is applied to the section itself, and that the road allowance retains its width of 1 chain or 1½ chains, with the exception of the roads on correction lines, which are subject to a widening or narrowing as hereinafter explained.

TABLE VIII.

Chord Azimuths, Jogs, &c., for Correction Lines, Third and Fourth Systems of Survey.

This table gives for the third and fourth systems the same quantities as are given in Table VII for the first and second systems.

The surplus or deficiency is in all cases divided equally among all the quarter sections.

TABLE IX.

Latitudes, and Widths in Chains, of Northern Boundaries of Sections in First and Second Systems of Survey.

This table gives the latitudes in degrees and decimals of a degree for the northern boundaries of all sections in the first and second systems.

The sections numbered in the second column are those adjacent to the eastern boundary of the township. The latitudes of interior sections lying west of these are the same. Thus the northern boundaries of sections 14, 15, 16, 17 and 18 have the same latitude as the north boundary of 13, and so for the other east and west tiers of sections.

These latitudes are computed by converting the latitudes given in Table III into degrees and decimals, and interpolating for the intermediate lines.

The logarithmic secant and tangent of the latitude are given in the table for use in calculation of azimuth observations.

In the last column of the table are given the widths of the north boundaries of the quarter sections (in the second system of survey). These are calculated for the correction lines in the manner explained under Table VII, and for the intermediate lines by interpolation.

TABLE X.

Latitudes and Widths in Chains of Northern Boundaries of Sections in Third and Fourth Systems of Survey.

This table gives for the third system the same quantities as are given in Table IX for the first and second.

The table may also be applied to the fourth system by correcting the latitudes of the alternate section lines, viz., the north boundaries of sections 1, 13 and 25 in each township, by subtracting therefrom 0°·0001, the equivalent in arc of 50 links. The change in the logarithmic secant and tangent is inappreciable, as these logarithms are given to only five places of decimals. The widths of quarter sections in the last column must be increased by 50 links.

TABLE XI.

To Reduce Chains to Decimals of a Township Side.

This is a short table giving the equivalents of chained distances in terms of a township side, for township sides of the first and second systems (489 chains), for east and west lines of the third and fourth systems (486 chains) and for north and south lines of these last systems (483 chains). The table is useful in calculating the difference in azimuth of an east or west line between a township corner and any other point upon it, and for similar purposes.

TABLE XII.

Correction to Widths of Roads on Correction Lines on Account of Curvature.

The township corners on the north and south sides respectively of the road on correction lines lie on two circles of latitude, which are one and a-half chains apart in the first and second systems, and one chain apart in the third system. The township sides are chords of these circles, and therefore lie north of them.

Hence, since on account of the jog the township corners north and south of the road are not opposite to one another, the township side south of the road will pass the township corner north of the road at a distance less than the theoretical one chain; while the township side north of the road will pass the corner south of the road at a distance greater than one chain.

The correction to the width of the road on this account for various lengths of the jog, is given in the table. The width of the road at points other than the township corners, varies in proportion to the distance.

This table may be used where it is required to establish the posts on one side of a correction line, by offsets from the other side.

The calculation of the differences of width is made as described below for Table XIII, the difference being merely the offset from the township chord to the parallel.

In Table XII are also given corrections to the chord azimuths and deflection offsets on correction lines (given in Table VII), when the north side of the road allowance is surveyed instead of the south. The correction is small and of little importance in surveying, except in the case of the second system of survey, where the correction lines were surveyed instead of the base lines, as the basis of the townships, across four ranges before closing, and the azimuth was consequently of importance.

In the first system the correction line is surveyed across two ranges as a trial line, and afterwards corrected to the true line; and in the third system the correction line is only surveyed across one range at a time, and as a trial line. In these systems, therefore, the azimuth used in the survey is of little importance.

TABLE XIII.

Difference of Latitude between Township Corners and Section and Quarter Section Corners.

This table is used when it is required to find accurately the latitude of any point within a township, as when it is desired by connecting with an astronomically determined latitude point to find the error of the survey lines.

If A be the initial azimuth of the township chord, A^1 its azimuth at a distance x from the corner of the township, Φ the latitude of the township corner, Φ^1 the latitude of a point on the chord distant x from the corner.

Then by spherical trigonometry

$$\frac{\cos \Phi^1}{\cos \Phi} = \frac{\sin A}{\sin A^1}$$

whence

$$\tan \frac{\Phi^1 - \Phi}{2} \tan \frac{\Phi^1 + \Phi}{2} = \tan \frac{A^1 - A}{2} \cot \frac{A^1 + A}{2}$$

putting

$$A = \tfrac{1}{2}(\pi - \theta)$$
$$A^1 = \tfrac{1}{2}(\pi - \theta^1)$$

where θ and θ^1 are expressed in circular measure, and are very small, so that their cubes may be neglected. Also $\Phi^1 - \Phi$ is very small, and $\Phi^1 + \Phi$ is very nearly equal to 2Φ.

$$\text{Then } \Phi^1 - \Phi = \frac{\theta - \theta^1}{2} \frac{\theta + \theta^1}{4} \cot \Phi = \frac{\theta^2 - \theta_1{}^2}{8} \cot \Phi$$

and θ = convergence of meridians for one township chord;

$$\therefore \theta = \frac{c}{N} \tan \Phi, \; c \text{ being the length of the chord,}$$

and $\frac{\theta_1}{\theta} = \frac{c - 2x}{c}$, whence $\theta^2 - \theta_1^2 = \frac{4(c-x)x}{c^2} \theta^2$

Therefore

$$\Phi^1 - \Phi = \frac{(c-x)x}{2N^2} \tan \Phi$$

or difference of latitude in chains =

$$R(\Phi^1 - \Phi) = \frac{R}{2N^2} x(c-x) \tan \Phi$$

The computation has been made for the first system of survey, but may be used for any system without sensible error.

CHAPTER III.

PROBLEMS CONNECTED WITH THE SYSTEM OF SURVEY.

Correction for Height above Sea Level.

The tables have been calculated from the dimensions of the earth surface at sea level.

The township sides are actually measured on surfaces elevated above sea level, and therefore the differences of latitude and longitude calculated from the tables are greater than those actually covered by the township sides.

Any measured distance may be reduced to sea level by subtracting the correction $\frac{h}{r} x$, x being the distance, h the elevation above sea level, and r the radius of curvature of the line under consideration.

In general N (see Table I) can be used instead of r.

Base lines when the system of survey is exactly followed are established by direct measurement from the 49th parallel, northward along an initial meridian.

Hence the latitude of a base line should be less than that given in table by $(\psi - 49°) \frac{h}{R}$ where h is the mean elevation of the initial meridian between the 49th parallel and the base under consideration.

Many base lines, however, have been established, not by this direct measurement, but by the survey of township meridians exterior from other bases. If the actual latitudes of these base lines are required, account must be taken of the elevations of all the north and south lines through which the connection with the 49th parallel has been made. It is obvious, however, that the average elevation of the country above the sea will give a sufficiently accurate result, since the small errors due to difference of elevation are masked by errors of survey.

On the base lines the effect of elevation above sea level is to decrease the difference of longitude covered by one range, and this must be allowed for in establishing an initial meridian by means of chainage along a base line, or in estimating the accuracy of measurement of a base line by its closing on an initial meridian, since the initial meridians, except the first, have been placed on even degrees of longitude (every fourth degree).

The correction for elevation above sea level is, in latitude 51°, 0·00382 chains for one mile distance at an elevation of 1,000 feet, and varies directly as the elevation and distance. It changes somewhat with the latitude, but slightly, and the correction in any particular case may be taken as the same as that for latitude 51°. If extreme accuracy be required, the formula given above, $\frac{h}{r} x$ may be used.

The error in the length of township chords of course involves an error in deflection angles and azimuths, but this is too small to be appreciable.

Latitudes and Longitudes of Points in the System.

By "points in the system" I mean the corners of specified sections, or points referred to them by connecting lines. In the latter case the lines, if short, may be reduced to latitude and longitude by means of "latitude and departure" from a traverse table, and by using Table XVIII.

Thus, the problem is reduced to the determination of the latitude and longitude of any section corner.

Latitude.

The latitude of the section corner can be at once found by interpolation from Table III or Table IV, according as the section is in the first, second or third system.

It must be remembered that in the first and second systems, the section posts on a meridian are 81·50 chains apart, and that in the third system they are alternately 81 and 80 chains.

The latitude can also be taken directly from Table IX or X to the fourth decimal place of degrees.

Since the section corners are presumed to be at distances of even sections from the north and south boundaries of the township, being established by survey from those boundaries, the latitude found as above must, when the section corner is not on the meridian outline of the township, be increased by the correction given by Table XIII.

In the first system the sections are not measured on meridians from the north or south boundary of the township, but on lines parallel to the eastern boundary of

the township. Hence, theoretically the difference of latitude between the given corner and the township outline should be decreased in the ratio of cosine azimuth of the section line to unity; but this correction is practically insignificant. The correction for sea level may also be applied.

Longitude, Third System,

In the second and third systems the section lines are true meridians from the base line north and south two townships. Hence the longitude of a section corner is the same as that of the corresponding corner on the base line from which the township has been surveyed.

Then if $d\lambda$ be the longitude covered by one range on that base line, and if n be the number of the range in which the section lies, m the number of sections lying between the given section and the eastern boundary of the township, the number of ranges which intervene between the initial meridian and the eastern boundary of the given section is $n-1+\frac{m}{6}$, and the difference in longitude between it and the initial meridian is $\left(n-1+\frac{m}{6}\right)d\lambda$. This added to the longitude of the initial meridian gives the longitude of the eastern boundary of the section.

The longitude of the Principal or First Meridian is 97° 27′ 08″·4,

The longitudes of the Second, Third, Fourth, &c., Meridians are 102°, 106°, 110°, 114°, &c., subject to certain errors of survey, which cannot be discussed at present.

The difference of longitude should be corrected for height above sea if precision is required. This can be done by multiplying it by $\left(1-\frac{h}{N}\right)$

For example:

The N.E. corner of Sec. 16, Tp. 23, R. 17, W. of the Fourth Meridian (third system of survey). Here $n=17$, $m=3$, and the township is surveyed from the 7th base, for which we find from Table IV $d\lambda = 8'\ 22''{\cdot}411 = 502''{\cdot}411$. Therefore longitude of the section line

$$= 110° + (502''{\cdot}411 \times 16\tfrac{3}{6}) = 112°\ 18'\ 09''{\cdot}78.$$

The corner is three sections, *i.e.*, 242 chains north of the 5th correction line, and its latitude is therefore (from Table IV)

$$50°\ 34'\ 20''{\cdot}77 + 10'\ 28''{\cdot}88 \times \frac{242}{966} = 50°\ 34'\ 20''{\cdot}77 + 157''{\cdot}55 = 50°\ 36'\ 58{\cdot}32''.$$

Longitude, First System.

In the first system the procedure for the longitude is a little different. The section lines are drawn parallel to the east side of the township, so that the difference of longitude between the section line and the east boundary of the township is not the same as on the base line, but is equal to the actual distance from the boundary of the township divided by $P \sin 1''$, P sin″ being taken from Table I for the actual latitude of the section post. Thus using the same notation as before

Diff. of longitude from initial meridian $= (n-1)\,d\lambda + \frac{81{\cdot}50 \times m}{P \sin 1''}$, $d\lambda$ being taken from Table III (1st system) for the governing base line, or it may be calculated by the equivalent formula

$$\text{diff. of longitude} = \left(n-1+\frac{m}{6}\right)d\lambda + \frac{Q}{P \sin 1''}$$

where $Q = 2\,m\,(40-w)$, w being the width of quarter sections as taken from the last column of Table IX.

Longitude, Second and Fourth Systems.

Longitudes in the 2nd system are calculated in the same way as those in the 3rd, taking $d\lambda$ from Table III instead of Table IV. In the 4th system the process is the same, as for the 3rd system, and the same table is used—Table IV.

Effect of Errors of Survey.

An error in the latitude of the base line, or an error in the longitude of the initial meridian, of course increases or decreases by the amount of the error the latitude or longitude of the section corner. Similarly a chainage error on the base line affects the longitude directly. In the computation all known errors of this kind must be allowed for.

An error in the latitude of the base line also affects the longitude covered by 486 chains (or 489) chains measured along the base line, since 486 chains covers less longitude if the base line be moved north. The manner in which the effect of an error of this kind may be estimated will be best shown by an example.

Suppose the 6th base line (3rd system) to be placed 10 chains too far north, we find from Table IV

$d\lambda$, for 6th base line $= 498{\cdot}662$
$d\lambda$ for 6th correction line $= 500{\cdot}527$

The 6th correction line is two townships, *i.e.*, 966 chains north of the 6th base line, and the difference in $d\lambda$ for these lines is $1''{\cdot}865$. Therefore, $d\lambda$ for the actual position of the 6th base line, 10 chains north of its theoretical position, is

$$498''{\cdot}662 + 1''{\cdot}865 \times \frac{10}{966} = 498''{\cdot}681$$

The correction, in the case supposed, to $d\lambda$ for one range is $0''{\cdot}019$, and in 29 ranges, (about the distance apart of two initial meridians) it amounts to $0''{\cdot}019 \times 29 = 0''{\cdot}55$, or 54 links.

GIVEN THE LATITUDE AND LONGITUDE OF A POINT, TO FIND ITS POSITION WITH REGARD TO THE SURVEY SYSTEM, *i.e., to find in what section it is, and the township and range, and its distance from the N. E. corner of the section.*

Second, Third and Fourth Systems.

This is the converse of the preceding problem. The first step is to find, in the manner explained above, the latitude of the section line next north of the given latitude. The difference between these two latitudes is reduced to chains by Table I. This gives the distance (x) in chains to be measured from the point to find the north boundary of the section.

The number of sections by which the section line is north of the southern boundary of the township in which it lies is to be noted. Call this number a, and the number of the township t.

We also know the number of the nearest base line, *i.e.* the base line on which depends the survey of township t. From table IV we take out $d\lambda$ for this base line.

From the given longitude of the point subtract the longitude of the initial meridian. Divide the difference by $d\lambda$, with quotient n and remainder r. Divide r by $\frac{d\lambda}{6}$ with quotient b and remainder s. S reduced from seconds of longitude to chains by Table I, with argument, latitude of the given point, gives the distance (y) to be measured east from the point to find the eastern line of the section.

We now know that the given point is x chains south and y chains west of the north-east angle of some section in township No. t and range No. $(n + 1)$ west of the initial meridian; and also that the northern boundary of the section is a sections north of the southern boundary of the township, and that the eastern boundary is b sections west of the eastern boundary of the township.

It is now easy by means of a skeleton township diagram to determine the numbers of the section; *e.g.* if $a = 5$, $b = 3$, the section is 28.

Without a township diagram, the section number can be found from the formula

$$\text{No. of section} = \tfrac{1}{2}\left\{12a - 5 \pm (2b - 5)\right\}$$

The upper sign being taken when a is odd, and the lower when a is even. These two rules are comprised in the general formula

$$\text{No. of section} = \tfrac{1}{2}\left\{(12a - 5) - (-1)^a(2b - 5)\right\}$$

The calculation for the second system is the same as above, using the proper tables for that system. It is also the same for the fourth system.

In this manner have been computed the positions of a great many section corners in British Columbia (fourth system of survey) with reference to points along the line of the Canadian Pacific Railway, the latitudes and longitudes of these points having been first determined by a traverse survey.

First System of Survey.

The procedure in this system is the same as above, except that the total difference of longitude from the eastern boundary of the township (instead of the nearest section line) must be reduced to chains, and from the chain distance must be subtracted the nearest multiple of 81·50.

Fractional Township or Range between parts of the Country Surveyed under different Systems of Survey.

Townships of the first and second systems adjoin each other without overlap or deficiency, since the townships in these two systems are of the same dimensions. Similarly of the third and fourth systems.

But where townships surveyed under the latter systems abut on townships of the first or second system, a fractional township or range occurs. It is only necessary to consider the case of the third system abutting on the first or second, since the fourth does not occur in juxtaposition with these latter systems.

Fractional Township.

Townships of the third system are 6 chains shorter, measured north and south than the others. The townships in both cases are measured north from the 49th parallel, and hence the third system falls short of the other by 6 chains for each township, and the northern boundary of a township of the third system is therefore south of the northern boundary of the same township of the first or second system by 6 chains multiplied by the number of the township.

Thus the 5th correction line (Tp. 18), as surveyed under the third system, is $6 \times 18 = 108$ chains south of its position under the second system. For twelve ranges west of the Second Meridian, the territory from the 5th correction line northward to the 8th correction line was surveyed under the second system, while the country south of the former line has been surveyed under the third system. There is therefore an additional township (measuring 108 chains from north to south) lying between Township 18 of the third system and Township 19 of the second system. (This fractional township is called Township 19A, and is subdivided according to the third system. See Manual of Surveys.)

Fractional Range.

Townships of the third system are 3 chains narrower (measured east and west along the base line) than those of the first and second systems. The overlap of the latter systems over the third, however, is not equal to 3 chains multiplied by the number of ranges, but exceeds this, since the widths are laid off along base lines which lie in different latitudes, and hence the convergence of meridians comes into play.

The readiest method of calculating this overlap is as follows:—

Let $d\lambda_1$ be the longitude covered by one range of the base line in the first or second system as found from Table III.

Let $d\lambda$ be the same quantity for the base line of the third system (from Table IV).

Then $d\lambda_1 - d\lambda$ is the difference of the longitude between the exterior meridians of range one, as surveyed under the two systems.

The difference of longitude at the eastern boundary of the nth range will be

$$(n - 1)\ (d\lambda_1 - d\lambda)$$

This reduced to chains is

$$(n - 1)\ (d\lambda_1 - d\lambda)\ P \sin 1''$$

$P \sin 1''$ being taken from the proper table for the latitude of the base or section line on which the overlap is required.

Example.

The meridian outline between Ranges 12 and 13, west of the 2nd Meridian, from Township 19 to Township 22, inclusive, is the western boundary of a tract of country surveyed under the second system of survey. Required the width of Range 13, as surveyed under the third system, on the northern boundaries of Townships 19, 20, 21 and 22.

The base line on which this meridian outline is based, is the 6th base line, or northern boundary of Township 20.

From Table III, $d\lambda_1 = 8'\ 21''{\cdot}972$
do IV, $d\lambda = 8'\ 18''{\cdot}662$

whence $d\lambda_1 - d\lambda = 3''{\cdot}310$

and at the eastern boundary of the thirteenth range, the difference of longitude is $3{\cdot}310 \times 12 = 39''{\cdot}72$.

We have then for the northern boundary of Township 19 (third system):

Log. $39{\cdot}72 = 1{\cdot}5990092$
Table IV, Log. $P \sin 1'' = 9{\cdot}9896352$

$1{\cdot}5886444$
Nat. number $= 38{\cdot}783$

For the northern boundary of Township 20:

Log. $39{\cdot}72 = 1{\cdot}5990092$
Log. $P \sin 1'' = 9{\cdot}9888297$

$1{\cdot}5878389$
Nat. number $= 38{\cdot}711$

For the northern boundary of Township 21:

Log. $39{\cdot}72 = 1{\cdot}5990092$
Log. $P \sin 1'' = 9{\cdot}9880192$

$1{\cdot}5870284$
Nat. number $= 38{\cdot}639$

For the northern boundary of Township 22:

Log. $39{\cdot}72 = 1{\cdot}5990092$
Log. $P \sin 1'' = 9{\cdot}9872086$

$1{\cdot}5862178$
Nat. number $38{\cdot}567$

Hence the north boundaries of Townships 19, 20, 21 and 22, surveyed under the third system in Range 13, have their eastern tiers of section narrowed by 38·783, 38·711, 38·639 and 38,567, respectively.

Now, the full widths of these sections when regular is got from Table X, by multiplying the " width of quarter section " by 2.

Thus, the width of the eastern tier of sections in Range 13 are:

For Township 19,	80·15 — 38·78 =	41·37	chains.
do 20,	80· — 38·71 =	41·29	do
do 21,	79·85 — 38·64 =	41·21	do
do 22,	79·70 — 38·57 =	41·13	do

These widths must be increased by one chain for road, if the widths from post to post are required.

For the township lines to the north of the correction line, viz.: 23, 24, 25 and 26, the width of Range 13 may be found in the same way, using the $d\lambda$ from Tables III and IV for the seventh base instead of the sixth.

If the width of the section on the north side of the 6th correction line is required, that is, the south boundary of Township 23, it must be remembered that here, on account of the correction line being thrown south, from the less depth of the townships of the new system, the southern boundary of Township 23 of the third system, which is brought from the 7th base, intersects the second system south of the correction line, *i. e.* on a line brought from the 6th base.

Therefore we have

For the second system, Table III, $d\lambda_1$ 6th base = 8′ 21″ ·972

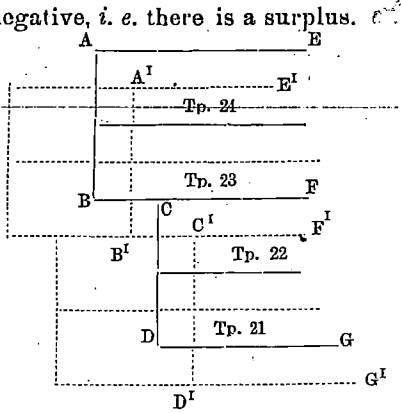
third do do IV, $d\lambda$ 7th do = 8′ 22″ ·411

$\therefore\ d\lambda_1 - d\lambda =$ — ·439

and for twelve ranges 12 $(d\lambda_1 - d\lambda) =$ — 5″ ·268

With the difference of longitude 5″ ·268 and the P sin 1″ for the 6th correction line, third system, we get the required jog.

It will be noticed that the overplus is negative, *i. e.* there is a surplus.



The heavy lines represent the second system, the dotted ones the third. The line A^1B^1 is the one which we have just considered; it falls to the east of AB, but to the west of CD.

The lines in the figure are all township lines. Thus it will be seen that there is a small piece of land, B^1 C, which is in fact a township of itself. Its designation would be Township 23 A, Range 12.

Second Example.

Required the depth, north and south, of Township 27, Range 19, west of the Principal Meridian.

The north boundary of Township 26 is the northern boundary of a tract of country surveyed under the first system.

Since each township of the third system is 6 chains shorter north and south than one of the first system, the northern boundary of Township 26 in the third system is 6 x 26 = 156 chains south of the same boundary under the first system.

Therefore the distance from the north boundary of Township 26, first system, to the north-east angle of Section 12, Township 27, third system, is 161 — 156 = 5 chains.

Since 1·50 chains must be allowed for road 3·50 chains is the available width of the strip of land.

FRACTIONAL SECTIONS ADJOINING AN INITIAL MERIDIAN.

The longitude of the Principal Meridian is 97° 27′ 08″·4.

That of the	2nd	Initial Meridian	102° 00′ 00″
"	3rd	"	106° 00′ 00″
"	4th	"	110° 00′ 00″
"	5th	"	114° 00′ 00″
"	6th	"	118° 00′ 00″
"	7th	"	122° 00′ 00″

These longitudes are subject to correction for errors of survey, of which it is intended to treat in a future paper.

For the present purpose we shall use the above longitudes.

The difference of longitude between the first meridian and the second is 4° 32′ 51″·6 = 16371″·6, and between the others successively 4° = 14400″.

The width of the last range in seconds on a given base line is got by subtracting from 16371″·6 or 14400 the nearest integral multiple of $d\lambda$ as given by Table III or IV (according to which system of survey is used). Thus for the width of the last range on the 5th base line between the 2nd and 3rd Initial Meridians (third system of survey) we have from Table IV $d\lambda = 494''{\cdot}988$ and we find, by dividing 14400 by 494·988, a quotient 29 with remainder 45·348. That is, the width of Range 30 on the 5th base, or the difference of longitude between the 3rd Initial Meridian and the meridian forming the eastern boundary of Townships 15, 16, 17 and 18, Range 30, west of the second Initial Meridian is 45″·348. This can be converted into chains by multiplying by log P sin 1″, taken from Table IV for the section line whose length is required—whether the southern boundary of Township 15, or the northern boundary of Townships 15, 16, 17 or 18, or any of the intermediate section lines.

If the width of the last broken section be required, and if the remainder, after tracting the integral multiple of $d\lambda$ is greater than one sixth of $d\lambda$, integral multiples of $\frac{1}{6}\,d\lambda$ (difference of longitude covered by one section on the base line) must be subtracted until the remainder is less than $\frac{1}{6}\,d\lambda$. This remainder may then be converted to chains by multiplying by P sin 1″ taken out of the Table for the latitude of the line under consideration. The reason for this is that the widths in seconds of longitude are the same for all sections from the base to the correction line (in the third system).

The result thus found should be corrected for the mean height of the base line above sea level, and also for any error in the positions of the 2nd and 3rd Meridians, relative to each other.

CHAPTER IV.

SOLUTIONS OF SOME PROBLEMS IN PRACTICAL GEODESY.

GIVEN THE LATITUDE AND LONGITUDE OF A POINT ON THE EARTH'S SURFACE, AND THE DISTANCE AND AZIMUTH THEREFROM OF A SECOND POINT, *required the latitude and longitude of the second point and the azimuth of the first point as seen from the second.*

The earth being considered a sphere, with radius equal to the normal at the place (N), the distance (K) may be reduced to arc by the formula

$$u'' = \frac{K}{N \sin 1''}$$

Then we have a spherical triangle formed by the two points and the north (or south) pole of the earth, the sides being the colatitudes of the points ($90° - \Phi$ and $90° - \Phi'$) and u''; and the angles being the azimuths counted from the north of the points from one another, and the difference of longitude. Any three of these parts being given, the triangle may be solved by the usual formulæ of spheric trigonometry.

Since, however, the side u'' is very small compared with the radius of the sphere, and therefore the triangle cannot be accurately solved without logarithms of many

decimal places, a more practical solution can be obtained by expanding the difference of latitude, &c., in series:—

We then have for distances not much exceeding 20 miles

$$\phi = \Phi + u' \cos A - (u'' \sin A)^2 \sin 1'' \tan \Phi$$
$$\lambda' = \lambda - (u'' \sin A) \sec \Phi'$$
$$A' = 180° + A + (u'' \sin A) \sec \Phi' \sin \tfrac{1}{2} (\Phi + \Phi')$$

Where Φ and λ are the latitude and longitude respectively of the first point
Φ' and λ' those of the second point
A the azimuth of the second as seen from the first
A' do first do second

Longitudes being counted towards the west, and azimuths from the *north through east* from 0° to 360°.

Correction for Spheroidal Figure.

The above formulæ are derived on the assumption that the earth is a sphere. The solution for the spheroid can be obtained by applying a correction to the difference of latitude. There is no correction necessary, to the order of approximation of the formulæ given above, to either the difference of longitude or the difference of azimuth.

The spherical solution being made on a sphere whose radius is equal to the normal (N) at the place, which is the radius of the great circle perpendicular to the meridian, while the latitude is measured along the meridian, whose radius of curvature is R, the difference of latitude found as above must be multiplied by $\frac{N}{R'}$ $= 1 + e^2 \cos^2 \Phi$ nearly, or in other words $\Phi' - \Phi$ must be numerically increased by $e^2 \cos^2 \Phi\ (\Phi' - \Phi)$.

The spheroidal formulæ then become

$$\Phi' = \Phi + u'' \cos A - (u'' \sin A)^2 \sin 1'' \tan \Phi + e^2 \cos^2 \Phi \left\{ u'' \cos A - (u'' \sin A)^2 \sin 1'' \tan \Phi \right\}$$
$$\lambda' = \lambda - (u'' \sin A) \sec \Phi'$$
$$A' = 180° + A + (u'' \sin A) \sec \Phi'' \sin \tfrac{1}{2} (\Phi + \Phi')$$

The values of $e^2 \cos^2 \Phi$ for different latitudes, are :—

Φ	$e^2 \cos^2 \Phi$	Φ	$e^2 \cos^2 \Phi$	Φ	$e^2 \cos^2 \Phi$	Φ	$e^2 \cos^2 \Phi$	Φ	$e^2 \cos^2 \Phi$
42	·000376	48	·000305	54	·000235	60	·000170	66	·000113
43	365	49	293	55	224	61	160	67	104
44	353	50	282	56	213	62	150	68	096
45	341	51	270	57	202	63	140	69	088
46	329	52	258	58	191	64	131	70	080
47	317	53	247	59	181	65	122		

More Accurate Formulæ for Long Distances.

The above formulæ serve for distances not greater than say twenty miles. For longer distances, up to one hundred miles, the formulæ are (*see* "Lee's Table and Formulæ, Professional Papers of the United States' Engineers; and United States' Coast and Geodetic Survey, 1875," Appendix No. 19)—

$$\Phi' - \Phi = KB \cos A - K^2 C \sin^2 A - (\delta \Phi)^2 D + K^2 h E \sin^2 A,$$
$$\lambda' - \lambda = \frac{K \sin A}{N'' \sin 1'' \cos \Phi'}$$
$$A' = 180° + A - (\lambda' - \lambda) \frac{\sin \tfrac{1}{2} (\Phi + \Phi')}{\cos \tfrac{1}{2} (\Phi' - \Phi)} + (\lambda' - \lambda)^3 F$$

Where K = the distance

$B = \frac{1}{R \sin 1''}$ for the latitude of the initial point,

$C = \frac{\tan \phi}{2 NR \sin 1''}$ do

$D = \frac{\frac{3}{2} e^2 \sin \Phi \cos \Phi \sin 1''}{(1 - e^2 \sin^2 \Phi)^{\frac{3}{2}}}$ do

$E = \frac{1 + 3 \tan^2 \phi}{6 N^2}$ do

$b = KB \cos A$ or the first term of the expression for difference of latitude.

$\delta \Phi$ is an approximate value of $\Psi' - \Phi$, computed from the first and second terms of the expression.

$N' \sin 1''$ is taken for the latitude of the terminal point.

$\log F$, for latitude $45° = 7 \cdot 840$; for latitude $50° = 7 \cdot 792$; for latitude $55° = 7 \cdot 723$.

$\log e^2 = 7 \cdot 8305006$

$\log \sin 1'' = 4 \cdot 6855749$

The computation can be made by means of Table I, but more conveniently by means of the tables of the values of B, C, D and E, which are given in the United States Coast Survey Appendix above named.

It is to be noted that in the formulæ given in that appendix, the azimuth is counted from the south through west, while in those I have given for the shorter distances it is counted from north through east, conformably to the general practice in Dominion Land surveys. Hence as A is increased by 180°, the sign of cos A and sin A is changed.

Formulæ in Terms of Rectangular Co-ordinates.

Suppose the latitude and longitude (Φ and λ) of one point to be known, and the second point to be referred to the first by rectangular co-ordinates, y in direction of the meridian and x perpendicular to it, y being positive when measured north from the first point, and x positive when measured west.

$$\text{Then } \Phi' = \Phi + \frac{y}{R \sin 1''} - \tfrac{1}{2} \sin 1'' \tan \Phi' \left(\frac{x}{N \sin 1''}\right)^2 \frac{N \sin 1''}{R \sin 1''},$$

$$\lambda' = \lambda + \left(\frac{x}{N \sin 1''}\right) \sec \Phi'$$

$$A' = 180° + A - \left(\frac{x}{N \sin 1''}\right) \tan \Phi'$$

The expression for Φ' contains Φ', the quantity sought, in the last term. The value of Φ' to be used in computing this term is the approximate value of Φ' obtained from the first two terms $\phi + \frac{y}{R \sin 1''}$.

These formulæ may be used for differences of latitude and longitude on a traverse survey consisting of a number of short lines.

The co-ordinates with reference to the meridian of one of the points may be computed by summing the "latitudes and departures" taken from an ordinary traverse table for the several courses.

GIVEN THE LATITUDES AND LONGITUDES OF TWO POINTS, *to find the length and direction of their joining line.*

Let Φ and Φ' be the latitudes.

λ and λ' be the longitudes.

Then $(\Phi' - \Phi)$ multiplied by the factor $e^2 \cos^2 \Phi$ given in the table on page —, is the correction to the latitude to reduce it from the spheroid to the sphere. Half

of this correction is to be applied to each latitude, in such direction as to bring them nearer together.

We then have, calling these corrected latitudes l and l', and $(\Phi' - \Phi)\, e^2 \cos^2\Phi = \beta$

$$l = \Phi + \frac{\beta}{2}$$

$$\tan A = \frac{-(\lambda' - \lambda)\cos e'}{l' - l - \frac{1}{2}\sin 1''\,(\lambda' - \lambda)^2 \cos^2 l' \tan l}$$

$$l' = \Phi' - \frac{\beta}{2}$$

$$K = -\frac{(\lambda' - \lambda)\cos l'}{\sin A} N \sin 1''$$

$$A' = 180° + A - (\lambda' - \lambda)\sin\frac{l + l'}{2}$$

$N \sin 1''$ should be taken for the mean latitude $\frac{\Phi + \Phi'}{2}$; so also $e^2 \cos^2 \Phi$, although the difference in this latter will be inappreciable unless the difference of latitude is great.

KNOWING THE LATITUDES AND THE AZIMUTH *of one point from the other, to find the distance.*

Calculate β and l and l' as in the last case.

Find the auxiliary angles θ and $\theta - u$ from the equations

$$\tan\theta = -\frac{\tan l}{\cos A}$$

$$\sin(\theta - u) = \frac{\sin l'}{\sin l}\sin\theta$$

Whence u is known

then $K = u\, N \sin 1''$.

That value of θ is to be taken which is less than 90°, *i. e.*, if $\tan\theta$ be positive (when $\cos A$ is negative) θ will be a positive angle less than 90°. If $\tan\theta$ be negative, θ will be a negative angle. In the latter case the formula

$$\sin(\theta - u) = \frac{\sin l'}{\sin l}\sin\theta$$

becomes $\sin(\theta + u) = \frac{\sin l'}{\sin l}\sin\theta$. θ in this last being taken positively.

GIVEN THE LATITUDE OF ONE POINT, THE AZIMUTH FROM THIS TO THE OTHER, AND THE DIFFERENCE OF LONGITUDE, *to find the distance.*

That is, given Φ, $\lambda' - \lambda$, and A to find Φ', A' and K.

Let $d\lambda$ be the difference of longitude. The auxiliary angle θ is computed by the formula

$$\tan\theta = -\sin l \tan A.$$

$$\text{and } \tan a' = \frac{\tan\Phi \sin(\theta - d\lambda)}{\sin\theta}$$

$$\beta = (a' - \Phi)\, e^2 \cos^2\tfrac{1}{2}(a' + \Phi)$$

$$\Phi' = a' + \beta,\ l = \Phi + \frac{\beta}{2},\ l' = \Phi' - \frac{\beta}{2}$$

$$K = -d\lambda \frac{\cos l'}{\sin A} N \sin 1''.$$

TRIGONOMETRICAL LEVELLING.

To find the elevation of one station above another by observation of the apparent altitude.

Let K represent the distance apart of the two stations, C the angle subtended by the arc joining the two stations at the earth's centre (*i.e.*, more properly at the centre of the curvature of the arc):

Let m = the coefficient of refraction.
dh = difference of height of the two stations.
S = radius of curvature of the arc joining the stations.
E = measured angle of elevation.

Then $C = \frac{K}{S \sin 1''}$

$$dh = \frac{K \sin \{E + (\frac{1}{2} - m)\ C\}}{\cos \{E + (1 - m)\ C\}}$$

S, the radius of curvature of the arc, is found from R and N, given the azimuth of the arc, in the manner explained under Table I, but for ordinary purposes

$N \sin 1''$ or $R \sin 1''$ may be used instead of $S \sin 1''$.

m varies in different places, being greater at the sea-coast than in the interior. It runs from about ·065 to about ·080. Where accuracy is required it must be found by observation in the locality, by the method of reciprocal zenith distances, or otherwise.

Taking its value at ·070, the above formula becomes:

$$dh = \frac{K \sin (E + 0{\cdot}43\ C)}{\cos (E + 0{\cdot}93\ C)}$$

If the angle observed be an angle of depression instead of elevation, we have calling the observed angle D:

$$dh = \frac{-K \sin (D - 0{\cdot}43\ C)}{\cos (D - 0{\cdot}93\ C)}$$

APPENDIX—TABLES.

TABLE I.—Radii of Curvature of Meridians and Parallels, &c.

Latitude.	log N sin 1″.	log P sin 1″.	log R sin 1″.	Chains in 1″.		Seconds in one Chain.		English Miles in a Degree.	
				Lat-itude.	Long-itude.	Lat-itude.	Long-itude.	Lat-itude.	Long-itude.
° ′						″	″		
42 00	0·1873775	0·0584510	0·1857461	1·5337	1·1441	0·6520	0·8741	69·02	51·48
42 10	3818	73144	7589	1·5338	1·1411	0·6520	0·8764	69·02	51·35
42 20	3860	61711	7717	1·5338	1·1381	0·6520	0·8787	69·02	51·21
42 30	3903	50212	7845	1·5339	1·1351	0·6520	0·8810	69·02	51·08
42 40	3946	38645	7973	1·5339	1·1320	0·6519	0·8834	69·03	50·94
42 50	3988	27009	8101	1·5339	1·1290	0·6519	0·8857	69·03	50·81
43 00	4031	15306	8230	1·5340	1·1260	0·6519	0·8881	69·03	50·67
43 10	4074	0·0503534	8358	1·5340	1·1229	0·6519	0·8905	69·03	50·53
43 20	4117	0·0491693	8487	1·5341	1·1199	0·6519	0·8930	69·03	50·39
43 30	4160	79782	8615	1·5341	1·1168	0·6518	0·8954	69·04	50·26
43 40	4203	67802	8744	1·5342	1 1137	0·6518	0·8979	69·04	50·12
43 50	4245	55750	8872	1·5342	1·1106	0·6518	0·9004	69·04	49·98
44 00	4288	43629	9001	1·5343	1·1075	0·6518	0·9029	69·04	49·84
44 10	4331	31437	9129	1·5343	1·1044	0·6518	0·9054	69·04	49·70
44 20	4374	19173	9258	1·5344	1·1013	0·6517	0·9080	69·05	49·56
44 30	4417	0·0406838	9387	1·5344	1 0982	0·6517	0·9106	69·05	49·42
44 40	4460	0·0394430	9515	1·5344	1·0951	0·6517	0·9132	69·05	49·28
44 50	4503	81949	9644	1·5345	1·0919	0·6517	0·9158	69·05	49·14
45 00	4546	69396	9773	1·5345	1·0888	0·6517	0·9185	69·05	49·00
45 10	4588	56768	0·1859901	1·5346	1·0856	0·6516	0·9211	69·06	48·85
45 20	4631	44067	0·1860030	1·5346	1·0824	0·6516	0·9238	69·06	48·71
45 30	4674	31292	0159	1·5347	1·0793	0·6516	0·9266	69·06	48·57
45 40	4717	18442	0288	1·5347	1·0761	0·6516	0·9293	69·06	48·42
45 50	4760	0·0305517	0416	1·5348	1·0729	0·6516	0·9321	69·06	48·28
46 00	4803	0·0292516	0545	1·5348	1·0697	0·6515	0·9349	69·07	48·14
46 10	4846	79439	0673	1·5349	1·0665	0·6515	0·9377	69·07	47·99
46 20	4889	66285	0802	1·5349	1·0632	0·6515	0·9405	69·07	47·85
46 30	4932	53054	0931	1·5349	1·0600	0·6515	0·9434	69·07	47·70
46 40	4974	39745	1059	1·5350	1·0568	0·6515	0·9463	69·07	47·55
46 50	5017	26358	1188	1·5350	1·0535	0·6515	0·9492	69·08	47·41
47 00	5060	0·0212893	1316	1·5351	1·0502	0·6514	0·9522	69·08	47·26
47 10	5103	0·0199349	1445	1·5351	1 0470	0·6514	0·9551	69·08	47·11
47 20	5146	85726	1573	1·5352	1·0437	0·6514	0·9581	69·08	46·97
47 30	5188	72021	1701	1·5352	1·0404	0·6514	0·9612	69·08	46·82
47 40	5231	58237	1829	1·5353	1·0371	0·6514	0·9642	69·09	46·67
47 50	5274	44372	1957	1·5353	1·0338	0·6513	0·9673	69·09	46·52
48 00	5316	30425	2085	1·5354	1·0305	0·6513	0·9704	69 09	46·37
48 10	5359	16396	2214	1·5354	1·0272	0·6513	0·9736	69·09	46·22
48 20	5402	0·0102285	2341	1·5354	1·0238	0·6513	0·9767	69·09	46·07
48 30	5444	0·0088090	2469	1·5355	1·0205	0·6513	0·9799	69·10	45·92
48 40	5487	73812	2598	1·5355	1·0171	0·6512	0·9831	69·10	45·77
48 50	5530	59449	2725	1·5356	1·0138	0·6512	0·9864	69·10	45·62
49 00	5572	45001	2852	1·5356	1·0104	0·6512	0·9897	69·10	45·47
49 10	5615	30469	2980	1·5357	1·0070	0·6512	0·9930	69·11	45·32
49 20	5657	15849	3106	1·5357	1·0037	0·6512	0·9964	69·11	45·16
49 30	5699	0·0001143	3234	1·5358	1·0003	0·6511	0·9998	69·11	45·01
49 40	5742	9·9986351	3361	1·5358	0·9969	0·6511	1·0031	69·11	44·86
49 50	5784	71470	3488	1·5358	0·9935	0·6511	1·0066	69·11	44·71
50 00	5826	56501	3615	1·5359	0·9900	0·6511	1·0101	69·12	44·55
50 10	5869	41444	3742	1·5359	0·9866	0·6511	1·0136	69·12	44·40
50 20	5911	26296	3870	1·5360	0·9832	0·6510	1·0171	69·12	44·24
50 30	5953	9·9911058	3995	1·5360	0·9797	0·6510	1·0207	69·12	44·09
50 40	5995	9·9895730	4122	1·5361	0·9763	0·6510	1·0243	69·12	43·93
50 50	6037	80309	4248	1·5361	0·9728	0·6510	1·0279	69·13	43·78
51 00	6079	64797	4374	1·5362	0·9693	0·6510	1·0316	69·13	43·62
51 10	6121	49192	4500	1·5362	0·9659	0·6510	1·0353	69·13	43·46
51 20	6163	33493	4626	1·5363	0·9624	0·6509	1·0391	69·13	43·31
51 30	6205	17701	4751	1·5363	0·9589	0·6509	1·0429	60·13	43·15
51 40	6247	9·9801813	4877	1·5363	0·9554	0·6509	1·0467	69·14	42·99
51 50	6289	9·9785830	5002	1·5364	0·9519	0·6509	1·0506	69·14	42·83
52 00	6330	69750	5127	1·5364	0·9484	0·6509	1·0544	69·14	42·68

APPENDIX—TABLES.

TABLE I.—Radii of Curvature of Meridians and Parallels, &c.

Latitude.	log N sin 1″.	log P sin 1″.	log R sin 1″.	Chains in 1″.		Seconds in one Chain.		English Miles in a Degree.	
				Lat-itude.	Long-itude.	Lat-itude.	Long-itude.	Lat-itude.	Long-itude.
° ′						″	″		
52 10	0·1876372	9·9753574	0·1865252	1·5365	0·9448	0·6508	1·0584	69·14	42·52
52 20	6413	37299	5376	1·5365	0·9413	0·6508	1·0624	69·14	42·36
52 30	6455	20926	5501	1·5366	0·9378	0·6508	1·0664	69·15	42·20
52 40	6496	9·9704454	5625	1·5366	0·9342	0·6508	1·0704	69·15	42·04
52 50	6538	9·9687882	5749	1·5366	0·9307	0·6508	1·0745	69·15	41·88
53 00	6579	71208	5873	1·5367	0·9271	0·6507	1·0786	69·15	41·72
53 10	6620	54435	5997	1·5367	0·9235	0·6507	1·0828	69·15	41·56
53 20	6661	37558	6120	1·5368	0·9199	0·6507	1·0870	69·16	41·40
53 30	6703	20579	6244	1·5368	0·9163	0·6507	1·0913	69·16	41·24
53 40	6744	9·9603495	6367	1·5369	0·9127	0·6507	1·0956	66·16	41·07
53 50	6785	9·9586307	6490	1·5369	0·9091	0·6507	1·0999	69·16	40·91
54 00	6825	69012	6612	1·5370	0·0055	0·6506	1·1043	69·16	40·75
54 10	6866	51612	6735	1·5370	0·9019	0·6506	1·1088	69·16	40·59
54 20	6907	34104	6857	1·5370	0·8983	0·6506	1·1132	69·17	40·42
54 30	6948	9·9516488	6979	1·5371	0·8946	0·6506	1·1178	69·17	40·26
54 40	6988	9·9498764	7101	1·5371	0·8910	0·6506	1·1223	69·17	40·09
54 50	7029	80928	7222	1·5372	0·8873	0·6505	1·1270	69·17	39·93
55 00	7069	62982	7343	1·5372	0·8837	0·6505	1·1316	69·17	39·77
55 10	7109	44924	7464	1·5373	0·8800	0·6505	1·1363	69·18	39·60
55 20	7150	26754	7585	1·5373	0·8763	0·6505	1·1411	69·18	39·44
55 30	7190	9·9408470	7705	1·5373	0·8727	0·6505	1·1459	69·18	39·27
55 40	7230	9·9390072	7825	1·5374	0·8690	0·6505	1·1508	69·18	39·10
55 50	7270	71557	7945	1·5374	0·8653	0·6504	1·1557	69.18	38·94
56 00	7310	52927	8065	1·5375	0·8616	0·6504	1·1607	69·19	38·77
56 10	7349	34177	8184	1·5375	0·8579	0·6504	1·1657	69·19	38·60
56 20	7389	9·9315310	8304	1·5376	0·8541	0·6504	1·1708	69·19	38·44
56 40	7429	9·9296324	8422	1·5376	0·8504	0·6504	1·1759	69·19	38·27
56 40	7468	77218	8541	1·5376	0·8467	0·6503	1·1811	69·19	38·10
56 50	7508	57987	8659	1·5377	0·8429	0·6503	1·1863	69·20	37·93
57 00	7547	38635	8777	1·5377	0·8392	0·6503	1·1916	69·20	37·76
57 10	7586	9·9219158	8894	1·5378	0·8354	0·6503	1·1970	60·20	37·59
57 20	7625	9·9199557	9012	1·5378	0·8317	0·6503	1·2024	69·20	37·43
57 30	7664	79829	9128	1·5378	0·8279	0·6503	1·2079	69·20	37·26
57 40	7703	59974	9245	1·5379	0·8241	0·6502	1·2134	69·20	37·09
57 50	7742	39991	9361	1·5379	0·8203	0·6502	1·2190	69·21	36·92
58 00	7780	9·9119877	9477	1·5380	0·8166	0·6502	1·2247	69·21	36·75
58 10	7819	9·9099633	9593	1·5380	0·8128	0·6502	1·2304	69·21	36·57
58 20	7858	79257	9709	1·5381	0·8090	0·6502	1·2362	69·21	36·40
58 30	7896	58747	9824	1·5381	0·8051	0·6502	1·2420	69·21	36·23
58 40	7934	38102	0·1869938	1·5381	0·8013	0·6501	1·2479	69·22	36·06
58 50	7972	17321	0·1870052	1·5382	0·7975	0·6501	1·2539	69·22	35·89
59 00	8010	9·8996403	0167	1·5382	0·7937	0·6501	1·2600	69·22	35·72
59 10	8048	75347	0280	1·5383	0·7898	0·6501	1·2661	69·22	35·54
59 20	8086	54150	0393	1·5383	0·7860	0·6501	1·2723	69·22	35·37
59 30	8123	32812	0506	1·5383	0·7821	0·6501	1·2786	69·23	35·20
59 40	8161	9·8911331	0619	1·5384	0·7783	0·6500	1·2849	69·23	35·02
59 50	8198	9·8889706	0731	1·5384	0·7744	0·6500	1·2913	69·23	34·85
60 00	8236	67936	0843	1·5385	0·7705	0·6500	1·2978	69·23	34·67
60 10	8273	46018	0955	1·5385	0·7667	0·6500	1·3044	69·23	34·50
60 20	8310	23952	1066	1·5385	0·7628	0·6500	1·3110	69·23	34·32
60 30	8347	9·8801735	1176	1·5386	0·7589	0·6500	1·3177	69·24	34·15
60 40	8384	9·8779367	1287	1·5386	0·7550	0·6499	1·3245	69·24	33·97
60 50	8420	56845	1397	1 5386	0·7511	0·6499	1·3314	69·24	33·80
61 00	8457	34169	1506	1·5387	0·7472	0·6499	1·3384	69·24	33·62
61 10	8493	9·8711336	1615	1·5387	0·7432	0·6499	1·3454	69·24	33·45
61 20	8529	9·8688345	1724	1·5388	0·7393	0·6499	1·3526	69·24	33·27
61 30	8565	65194	1832	1·5388	0·7354	0·6499	1·3598	69·25	33·09
61 40	8601	41882	1940	1·5388	0·7315	0·6498	1·3671	69·25	32·92
61 50	8637	9·8618406	2048	1·5389	0·7275	0·6498	1·3745	69·25	32·74
62 00	8673	9·8594766	2155	1·5389	0·7236	0·6498	1·3820	69·25	32·56
62 10	8708	70958	2261	1·5390	0·7196	0·6498	1·3896	69·25	32·38

'arallels, &c.

ds in one hain.	English Miles in a Degree.	
Long-itude.	Lat-itude.	Long-itude.
″		
1·0584	69·14	42·52
1·0624	69·14	42·36
1·0664	69·15	42·20
1·0704	69·15	42·04
1·0745	69·15	41·88
1·0786	69·15	41·72
1·0828	69·15	41·56
1·0870	69·16	41·40
1·0913	69·16	41·24
1·0956	66·16	41·07
1·0999	69·16	40·91
1·1043	69·16	40·75
1·1088	69·16	40·59
1·1132	69·17	40·42
1·1178	69·17	40·26
1·1223	69·17	40·09
1·1270	69·17	39·93
1·1316	69·17	39·77
1·1363	69·18	39·60
1·1411	69·18	39·44
1·1459	69·18	39·27
1·[illegible]	69·18	39·10
1·[illegible]	69.18	38·94
1·[illegible]	69·19	38·77
1·1657	69·19	38·60
1·1708	69·19	38·44
1·1759	69·19	38·27
1·1811	69·19	38·10
1·1863	69·20	37·93
1·1916	69·20	37·76
1·1970	60·20	37·59
1·2024	69·20	37·43
1·2079	69·20	37·26
1·2134	69·20	37·09
1·2190	69·21	36·92
1·2247	69·21	36·75
1·2304	69·21	36·57
1·2362	69·21	36·40
1·2420	69·21	36·23
·2479	69·22	36·06
·2539	69·22	35·89
·2600	69·22	35·72
·2661	69·22	35·54
·2723	69·22	35·37
·2786	69·23	35·20
·2849	69·23	35·02
·2913	69·23	34·85
·2978	69·23	34·67
·3044	69·23	34·50
·3110	69·23	34·32
3177	69·24	34·15
3245	69·24	33·97
3314	69·24	33·80
3384	69·24	33·62
3454	69·24	33·45
3526	69·24	33·27
3598	69·25	33·09
3671	69·25	32·92
3745	69·25	32·74
3820	69·25	32·56
3896	69·25	32·38

APPENDIX—TABLES.

TABLE I.—Radii of Curvature of Meridians and Parallels, &c.—*Concluded.*

Latitude.	log N sin 1″.	log P sin 1″.	log R sin 1″.	Chains in 1″.		Seconds in one Chain.		English Miles in a Degree.	
				Lati-tude.	Long-itude.	Lati-tude.	Long-itude.	Lati-tude.	Long-itude.
° ′						″	″		
62 20	0·1878744	9·8546982	0·1872368	1·5390	0·7156	0·6498	1·3973	69·25	32·20
62 30	8779	9·8522835	2474	1·5390	0·7117	0·6498	1·4051	69·26	32·03
62 40	8814	9·8498516	2579	1·5391	0·7077	0·6497	1·4130	69·26	31·85
62 50	8849	74022	2684	1·5391	0·7037	0·6497	1·4210	69·26	31·67
63 00	8884	49352	2789	1·5391	0·6997	0·6497	1·4291	69·26	31·49
63 10	8919	9·8424503	2893	1·5392	0·6957	0·6497	1·4373	69 26	31·31
63 20	8954	9·8399475	2997	1·5392	0·6917	0·6497	1·4456	69·26	31·13
63 30	8988	74262	3099	1·5393	0·6877	0·6497	1·4540	69·27	30·95
63 40	9022	48866	3202	1·5393	0·6837	0·6497	1·4626	69·27	30·77
63 50	9056	9·8323288	3305	1·5393	0·6797	0·6496	1·4712	69·27	30·59
64 00	9090	9·8297512	3407	1·5394	0·6757	0·6496	1·4800	69·27	30·41
64 10	9124	71546	3508	1·5394	0·6717	0·6496	1·4888	69·27	30·23
64 20	9158	45389	3609	1·5394	0·6676	0·6496	1·4978	69·27	30·04
64 30	9191	9·8219035	3709	1·5395	0·6636	0·6496	1·5069	69·28	29·86
64 40	9224	9·8192482	2809	1·5395	0·6596	0·6496	1·5162	69·28	29·68
64 50	9258	65730	3909	1·5395	0·6555	0·6495	1·5256	69·28	29·50
65 00	9291	38774	4008	1·5396	0·6514	0·6495	1·5351	69·28	29·32
65 10	9323	9·8111610	4106	1·5396	0·6474	0·6495	1·5447	69·28	29·13
65 20	9356	9·8084240	4205	1·5396	0·6433	0·6495	1·5544	69·28	28·95
65 30	9389	56659	4302	1·5397	0·6392	0·6495	1·5644	69·29	28·77
65 40	9421	28862	4399	1·5397	0·6352	0·6495	1·5744	69·29	28.58
65 50	9453	9·8000850	4496	1·5397	0·6311	0·6494	1·5846	69·29	28·40
66 00	9485	9·7972618	4592	1·5398	0·6270	0.6494	1·5949	69·29	28·21
66 10	9517	44164	4688	1·5398	0·6229	0·6494	1·6054	69·29	28·03
66 20	9549	9·7915485	4783	1·5398	0·6188	0·6494	1·6160	69·29	27·85
66 30	9580	9·7886577	4877	1·5399	0·6147	0·6494	1·6268	69·29	27·66
66 40	9612	57439	4972	1·5399	0·6106	0·6494	1·6378	69·30	27·48
66 50	9643	9·7828065	5065	1·5399	0·6065	0·6494	1·6489	69·30	27·29
67 00	9674	9·7798454	5158	1·5400	0·6023	0·6494	1·6602	69·30	27·11
67 10	9705	68602	5250	1·5400	9·5982	0·6493	1·6716	69·30	26·92
67 20	9735	38506	5342	1·5400	0.5941	0·6493	1·6833	69·30	26·73
67 30	9766	9·7708163	5434	1·5401	0·5900	0·6493	1·6951	69·30	26·55
67 40	9796	9·7677563	5525	1·5401	0·5858	0·6493	1·7070	69·31	26·36
67 50	9826	46718	5615	1·5401	0·5817	0.6493	1·7192	69·31	26·17
68 00	9856	9·7615610	5705	1·5402	0·5775	0·6493	1·7316	69·31	25·99
68 10	9886	9·7584241	5795	1·5402	0·5734	0·6493	1·7441	69·31	25·80
68 20	9916	52605	5883	4·5402	0·5692	0·6492	1·7569	69·31	25·61
68 30	9945	9·7520699	5972	1·5403	0·5650	0·6492	1·7698	69·31	25·43
68 40	0·1879974	9·7488520	6059	1·5403	0·5609	0·6492	1 7830	69·31	25·24
68 50	0·1880004	56064	6147	1·5403	0·5567	0·6492	1·7964	69·31	25·05
69 00	0032	9·7423324	6233	1·5404	0·5525	0·6492	1·8100	69·32	24·86
69 10	0061	9·7390298	6319	1·5404	0·5483	0·6402	1·8238	69·32	24·67
69 20	0090	56983	6405	1·5404	0·5441	0·6492	1·8378	69·32	24·49
69 30	0118	9·7323371	6490	1·5405	0·5399	0·6492	1·8521	69·32	24·30
69 40	0146	9·7289460	6574	1·5405	0·5357	0·6491	1·8666	69·32	24·11
69 50	0174	55244	6658	1·5405	0·5315	0·6491	1·8814	69·32	23·92
70 00	0202	9·7220719	6741	1·5405	0·5273	0·6491	1·8964	69·32	23·73

TABLE II.

CORRECTIONS to be applied to the logarithms of R sin 1″ and N sin 1″ in Table I, for Clarke's later values of the dimensions of the earth.

Latitude.	d (log R sin 1″).	d (log N sin 1″).	Latitude.	d (log R sin 1″).	d (log N sin 1″).
°			°		
42	−0·0000021	+0·0000063	56	+0·0000034	+0·0000081
43	17	64	57	37	82
44	13	66	58	41	84
45	09	67	59	45	85
46	05	68	60	48	86
47	−0·0000001	70	61	51	87
48	+0·0000003	71	62	55	88
49	07	72	63	58	89
50	11	74	64	61	90
51	15	75	65	64	91
52	19	76	66	67	93
53	23	77	67	70	93
54	26	79	68	73	94
55	30	80	69	76	95
			70	78	96

TABLE III.

LATITUDES, &c., of Base and Correction Lines. 1st and 2nd Systems of Surveys.

No. of Township.	Number of Line.	Latitude.	Log. N sin 1″.	Log. P sin 1″.	Log. R sin 1″.	Longitude covered by 489 Chains of westing.
		° ′ ″				′ ″
0	1st Base	49 00 00·00	0·1875572	0·0045001	0·1862852	8 03·959
2	Correction	10 36·86	5618	0·0029573	2989	05·681
4	2nd Base	21 13·70	5662	0·0014047	3122	07·421
6	Correction	31 50·52	5707	9·9998425	3256	09·177
8	3rd Base	42 27·33	5751	9·9982704	3391	10 951
10	3rd Correction	49 53 04·12	0·1875797	9·9966886	0·1863527	8 12·743
12	4th Base	50 03 40·89	5842	9·9950968	3662	14·552
14	Correction	14 17·64	5887	9·9334951	3797	16·379
16	5th Base	24 54·37	5932	9·9918831	3931	18·225
18	Correction	35 31·08	5976	9·9902611	4064	20.089
20	6th Base	50 46 07·77	0·1876021	9·9886289	0·1864198	8 21·972
22	Correction	56 44·44	6065	9·9869863	4331	23·875
24	7th Base	51 07 21·09	6110	9·9853334	4466	25·796
26	Correction	17 57·72	6154	9·9836700	4599	27·737
28	8th Base	28 34·33	6199	9·9819961	4733	29·698
30	8th Correction	51 39 10·92	0·1876243	9·9803116	0·1864867	8 31·678
32	9th Base	49 47·49	6287	9·9786163	4998	33·680
34	Correction	52 00 24·04	6332	9·9769104	5131	35·701
36	10th Base	11 00·57	6376	9·9751934	5264	37·744
38	Correction	21 37·08	6420	9·9734657	5395	39·808
40	11th Base	52 32 13·57	0·1876464	9·9717267	0·1865529	8 41·894
42	Correction	42 50·04	6508	9·9699768	5661	44·001
44	12th Base	53 26·49	6552	9·9682156	5791	46·130
46	Correction	53 04 02·92	6595	9·9664429	5920	48·282
48	13th Base	14 39·33	6640	9·9646592	6055	50·456

. N sin 1″ in Table I,
e earth.

in 1″).	d (log N sin 1″).
00034	+0·0000081
37	82
41	84
45	85
48	86
51	87
55	88
58	89
61	90
64	91
67	93
70	93
73	94
76	95
78	96

ystems of Surveys.

Log in 1″.	Longitude covered by 489 Chains of westing.
	′ ″
1862852	8 03·959
2989	05·681
3122	07·421
3256	09·177
3391	10 951
1863527	8 12·743
3662	14·552
3797	16·379
3931	18·225
4064	20.089
1864198	8 21·972
4331	23·875
4466	25·796
4599	27·737
4733	29·698
864867	8 31·678
4998	33·680
5131	35·701
5264	37·744
5395	39·808
865529	8 41·894
5661	44·001
5791	46·130
5920	48·282
6055	50·456

TABLE IV.

LATITUDES, &c., of Base and Correction Lines.

(Third System of Survey.)

Number of Township.	Name of Line.	Latitude.	Log. N sin 1″.	Log. P sin 1″.	Log. R sin 1″.	Longitude covered by 486 Chains.
		° ′ ″				′ ″
0	1st Base	49 00 00·00	0·1875572	0·0045001	0·1862852	8 00·990
2	Correction	10 29·05	5617	0·0029764	2987	02·681
4	2nd Base	20 58·07	5661	0·0014431	3119	04·388
6	Correction	31 27·08	5705	9·9999003	3251	06·112
8	3rd Base	41 56·08	5749	9·9983480	3383	07·852
10	3rd Correction	52 25·05	5794	9·9967861	3518	09·610
12	4th Base	50 02 54·01	5838	9·9952143	3650	11·385
14	Correction	13 22·96	5883	9·9936329	3786	13·178
16	5th Base	23 51·88	5927	9·9920418	3918	14·988
18	Correction	34 20·77	5971	9·9904407	4050	16·816
20	6th Base	44 49·65	6015	9·9888297	4182	18·662
22	Correction	55 18·51	6059	9·9872086	4314	20·527
24	7th Base	51 05 47·35	6103	9·9855774	4446	22·411
26	Correction	16 16·17	6147	0·9839365	4578	24·313
28	8th Base	26 44·98	6191	9·9822842	4710	26·235
30	8th Correction	37 13·76	6235	9·9806224	4842	28·176
32	9th Base	47 42·53	6279	9·9789500	4974	30·136
34	Correction	58 11·26	6322	9·9772671	5103	32·117
36	10th Base	52 08 39·98	6366	9·9755737	5235	34·118
38	Correction	19 08·69	6409	9·9738694	5364	36·139
40	11th Base	29 37·37	6453	9·9721545	5496	38·181
42	Correction	40 06·04	6497	8·9704288	5628	40·245
44	12th Base	50 34·69	6540	9·9686921	5757	42·329
46	Correction	53 01 03·31	6582	9·9669442	5883	44·436
48	13th Base	11 31·92	6626	9·9651855	6015	46·564
50	13th Correction	22 00·52	6670	9·9634156	6147	48·714
52	14th Base	32 29·09	6712	9·9616342	6273	50·887
54	Correction	42 57·65	6756	9·9598417	6405	53·083
56	15th Base	53 26·19	6799	9·9580375	6534	55·302
58	Correction	54 03 54·71	6841	9·9562218	6660	57·545
60	16th Base	14 23·21	6884	9·9543945	6789	8 59·811
62	Correction	24 51·69	6927	9·9525554	6918	9 02·102
64	17th Base	35 20·15	6969	9·9507044	7044	04·417
66	Correction	45 48·59	7012	9·9488415	7173	06·758
68	18th Base	56 17·01	7054	9·9469665	7298	09·123
70	18th Correction	55 06 45·42	7096	9·9450792	7424	11·515
72	19th Base	17 13·82	7139	9·9431798	7553	13·932
74	Correction	27 42·20	7181	9·9412680	7679	16·376
76	20th Base	38 10·55	7223	9·9393437	7805	18·847
78	Correction	48 38·89	7264	9·9374066	7928	21·345
80	21st Base.	59 07·20	7305	9·9354569	8051	23·871
82	Correction	56 09 35·49	7347	9·9334945	8177	26·424
84	22nd Base	20 03·77	7390	9·9315192	8306	29·006
86	Correction	30 32·03	7431	9·9295307	8429	31·618
88	23rd Base	41 00·28	7472	9·9275290	8552	34·258
90	23rd Correction	51 28·51	7513	9·9255140	8675	36·929
92	24th Base	57 01 56·70	7554	9·9234856	8798	39·630
94	Correction	12 24·89	7595	9·9214436	8921	42·362
96	25th Base	22 53·07	7637	9·9193880	9047	45·125
98	Correction	33 21·22	7678	9·9173186	9170	47·919
100	26th Base	43 49·36	7718	9·9152351	9290	50·747
102	Correction	54 17·48	7759	9·9131376	9413	53·607
104	27th Base	58 04 45·57	7799	9·9110259	9533	56·500

TABLE IV—*Concluded.*

LATITUDE, &c., of Base and Correction Lines—*Concluded.*

(Third System of Survey.)

Number of Township.	Name of Line.	Latitude.	Log. N sin 1″.	Log. P sin 1″.	Log. R sin 1″.	Longitude covered by 486 Chains.
		° ′ ″				′ ″
106	Correction	58 15 13·66	0·1877839	9·9088998	0·1869653	9 59·427
108	28th Base	25 41·73	7879	9·9067591	9773	10 02·389
110	28th Correction	36 09·78	7919	9·9046039	0·1869893	05.386
112	29th Base	46 37·81	7959	9·9024339	0·1870013	08·418
114	Correction	57 05·83	7999	9·9002490	0133	11·487
116	30th Base	59 07 33·83	8039	9·8980490	0253	14·593
118	Correction	18 01·81	8078	9·8958337	0370	17·735
120	31st Base	28 29·77	8117	9·8936029	0487	20·917
122	Correction	38 57·71	8157	9·8913568	0607	24·136
124	32nd Base	49 25·64	8196	9·8890948	0724	27·396
126	Correction	59 53·55	0·1878235	9·8868170	0·1870840	10 30·695

TABLE V.

CHORD AZIMUTHS, Deflections, Deflection Offsets, &c., for Base Lines.

(First and Second Systems of Survey.)

Number of Base Line.	Chord Azimuth.	Chord Azimuth.	Deflection.	Deflection.	Deflection Offset for 1 Chain Distance.	Longitude covered by 1 Range.	Number of Township.
	° ′ ″	°	′ ″	°	Inches.	s.	
1	89 56 57·4	89·9493	6 05·2	0·1014	1·402	32·3	0
2	55·1	·9486	09·8	·1027	1·420	32 5	4
3	52·8	·9480	14·5	·1040	1·438	32·7	8
4	50·4	·9473	19·2	·1053	1·456	33·0	12
5	48·0	·9467	24·0	·1067	1·474	33·2	16
6	89 56 45·6	89·9460	6 28·8	0·1080	1·493	33·5	20
7	43·1	·9453	33·8	·1094	1·512	33·7	24
8	40·6	·9446	38·8	·1108	1·531	34·0	28
9	38·1	·9439	43·8	·1122	1·551	34·2	32
10	35·5	·9432	49·0	·1136	1·570	34·5	36
11	89 56 32·9	89·9425	6 54·3	0·1151	1·591	38·8	40
12	30·2	·9417	59·6	·1165	1·611	35·1	44
13	27·5	·9410	7 05·0	·1180	1·632	35·4	48

–*Concluded.*

Log. R sin 1″.	Longitude covered by 486 Chains.
	′ ″
0·1869653	9 59·427
9773	10 02·389
0·1869893	05.386
0·1870013	08·418
0133	11·487
0253	14·593
0370	17·735
0487	20·917
0607	24·136
0724	27·396
0·1870840	10 30·695

for Base Lines.

ction for 1 ain ance.	Longitude covered by 1 Range.	Number of Township.
nches.	s.	
·402	32·3	0
·420	32 5	4
·438	32·7	8
·456	33·0	12
·474	33·2	16
·493	33·5	20
·512	33·7	24
·531	34·0	28
·551	34·2	32
·570	34·5	36
·591	38·8	40
·611	35·1	44
·632	35·4	48

TABLE VI.

CHORD AZIMUTHS, Deflections, Deflection Offsets, &c., for Base Lines.

(Third System of Survey.)

Number of Base Line.	Chord Azimuth Sexagesimal.	Chord Azimuth Decimal.	Deflection Sexagesimal	Deflection Decimal.	Deflection Offset for 1 Chain Distance.	Longitude covered by 1 Range.	Number of Township.
	° ′ ″	°	′ ″	°	Inches.	s.	
1	89 56 58·5	89·9496	6 03·0	0·1008	1·394	32·1	0
2	56·3	·9490	07·5	·1021	1·411	32·3	4
3	54·0	·9483	12·0	·1033	1·429	32·5	8
4	51·7	·9477	16·6	·1046	1·447	32·8	12
5	49·4	·9471	21·3	·1059	1·465	33·0	16
6	47·0	·9464	26·1	·1072	1·483	33·2	20
7	44·6	·9457	30·9	·1086	1·501	33·5	24
8	42·1	·9450	35·8	·1099	1·520	33·7	28
9	39·6	·9443	40·8	·1113	1·539	34·0	32
10	37·1	·9436	45·9	·1127	1·558	34·3	36
11	34·5	·9429	51·0	·1142	1·578	34·5	40
12	31·9	·9422	56·2	·1156	1·598	34·8	44
13	29·3	·9415	7 01·5	·1171	1·619	35·1	48
14	26·6	·9407	06·9	·1186	1·639	35·4	52
15	23·8	·9399	12·4	·1201	1·660	35·7	56
16	21·0	·9392	18·0	·1217	1·682	36 0	60
17	18·2	·9384	23·7	·1232	1·704	36·3	64
18	15·3	·9376	29·4	·1248	1·726	36·6	68
19	12·4	·9368	35·3	·1265	1·749	36·9	72
20	09·4	·9359	41·3	·1281	1·772	37·3	76
21	06·3	·9351	47·4	·1298	1·795	37·6	80
22	03·2	·9342	53·6	·1316	1·819	37·9	84
23	00 1	·9335	59·8	·1333	1·843	38·3	88
24	89 55 56·9	·9325	8 06·3	·1351	1·867	38·6	92
25	53·6	·9316	12·8	·1369	1·892	39·0	96
26	50·3	·9306	19·5	·1387	1·918	39·4	100
27	46·8	·9297	26·3	·1406	1·944	39·8	104
28	43·4	·9287	33·3	·1426	1·971	40·2	108
29	39·9	·9277	40·3	·1445	2·998	40·6	112
30	36·2	·9267	47·6	·1465	2·026	41·0	116
31	32·6	·9257	54·9	·1486	2·054	41·4	120
32	28·8	·9247	9 02·4	·1507	2·083	41·8	124

TABLE VII.

CHORD AZIMUTHS, Deflections, Deflection Offsets, Jogs, &c., for Correction Lines.

(First and Second Systems of Survey.)

Number of Correction Line.	Chord Azimuth.	Chord Azimuth.	Deflection.	Deflection.	Deflection Offset for one chain distance.	Length of one Range on Correction Line.		Jog.	Convergence or Divergence on half Section.	Number of Township.
						North side of Road.	South side of Road.			
	° ′ ″	°	′ ″	°	in inches	chains.	chains.	chains.	links.	
1	89 56 56·9	89·9491	6 06·2	0·1017	1·406	490·751	487·266	3·485	14·5	2
2	54·6	·9485	10·8	·1030	1·424	·773	·244	·529	14·7	6
3	52·3	·9479	15·5	·1043	1·442	·796	·222	·574	14·9	10
4	49·9	·9472	20·2	·1056	1·460	·818	·200	·618	15·1	14
5	47·5	·9465	25·0	·1069	1·478	·841	·177	·664	15·3	18
6	89 56 45·1	89·9459	6 29·8	0·1083	1·497	490·865	487·154	3·711	15·5	22
7	42·7	·9452	34·7	·1096	1·516	·888	·131	·758	15·7	26
8	40·2	·9445	39·7	·1110	1·535	·913	·107	·806	15·9	30
9	37·6	·9438	44·8	·1124	1·554	·937	·083	·854	16·1	34
10	35·0	·9430	50·0	·1139	1·574	·962	·058	·904	16·3	38
11	89 56 32·4	89·9423	6 55·2	0·1153	1·594	490·987	487·034	3·953	16·5	42
12	29·7	·9416	7 00·6	·1168	1·615	491·012	·008	4·004	16·7	46

r Correction Lines.

	Jog.	Convergence or Divergence on half Section.	Number of Township.
	chains.	links.	
6	3·485	14·5	2
4	·529	14·7	6
2	·574	14·9	10
0	·618	15·1	14
7	·664	15·3	18
4	3·711	15·5	22
1	·758	15·7	26
7	·806	15·9	30
3	·854	16·1	34
3	·904	16·3	38
1	3·953	16·5	42
5	4·004	16·7	46

TABLE VIII.

CHORD AZIMUTAS, Deflections, Deflection Offsets, Jogs, &c., for Correction Lines.

(Third System of Survey.)

Number of Correction Line.	Chord Azimuth Sexagesimal.	Chord Azimuth Decimal.	Deflection Sexagesimal.	Deflection Decimal.	Deflection Offset for one chain distance.	Length of one Range on Correction Line.		Jogs.	Convergence or Divergence on half section.	Number of Township.
						North side of Road.	South side of Road.			
	° ′ ″	°	′ ″	°	Inches.	chains.	chains.	chains.	chains.	
1	89 56 57·4	89·9493	7 05·2	0·1014	1·403	487·719	484·297	3·421	0·143	2
2	55·1	·9486	09·8	·1027	1·420	·740	·276	·463	·144	6
3	52·9	·9480	14·3	·1040	1·438	·762	·255	·507	·146	10
4	50·5	·9474	19·0	·1053	1·456	·784	·233	·551	·148	14
5	48·2	·9467	23·7	·1066	1·474	·806	·212	·594	·150	18
6	45·8	·9461	28·5	·1079	1·492	·829	·188	·641	·152	22
7	43·3	·9454	33·4	·1093	1·510	·852	·167	·685	·154	26
8	40·9	·9447	38·3	·1106	1·529	·875	·144	·731	·155	30
9	38·3	·9440	43·4	·1120	1·548	·899	·120	·779	·157	34
10	35·8	·9433	48·4	·1134	1·568	·923	·097	·826	·159	38
11	33·2	·9426	53·6	·1149	1·588	·947	·072	·875	·161	42
12	30·6	·9418	58·8	·1163	1·608	·972	·047	·925	·164	46
13	27·9	·9411	7 04·2	·1178	1·629	487·997	484·024	3·973	·166	50
14	25·2	·9403	09·6	·1193	1·650	488·023	483·998	4·025	·168	54
15	22·4	·9396	15·2	·1209	1·671	·049	·972	·077	·170	58
16	19·6	·9388	20·8	·1224	1·693	·075	·946	·129	·172	62
17	16·7	·9380	26·6	·1241	1·715	·102	·919	·183	·174	66
18	13·8	·9372	32·4	·1257	1·737	·130	·892	·238	·177	70
19	10·9	·9364	38·3	·1273	1·760	·158	·865	·293	·179	74
20	07·8	·9355	44·4	·1290	1·783	·187	·837	·350	·181	78
21	04·8	·9337	50·5	·1307	1 807	·215	·809	·406	·184	82
22	89 56 01·7	·9338	56·7	·1324	1·831	·245	·779	·466	·186	86
23	89 55 58·5	·9329	8 03·0	·1342	1·855	·275	·750	·525	·189	90
24	55·2	·9320	09·6	·1360	1·879	·306	·720	·586	·191	94
25	51·9	·9311	16·2	·1378	1·905	·338	·690	·648	·194	98
26	48·6	·9302	22·9	·1397	1·931	·369	·658	·711	·196	102
27	45 1	·9292	29·8	·1416	1·957	·402	·627	·775	·199	106
28	41·6	·9282	36·8	·1436	1·984	·434	·594	·840	·202	110
29	38·0	·9272	44·0	·1456	2·012	·469	·561	·908	·204	114
30	34·4	·9262	51·2	·1476	2·040	·503	·528	4·975	·207	118
31	30·7	·9252	58·6	·1496	2·068	·538	·493	5·045	·210	122
32	89 55 26·9	89·9241	9 06·2	·1517	2·097	488·574	483·458	5·116	·213	126

TABLE IX.

LATITUDE, with Logarithms of Secant and Tangent for the North Boundary of each Section, and the widths of Quarter Sections on such Boundaries.

(First and Second Systems of Survey.)

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	36	49°·0000	0·183 06		0·060 84		40·000
1	1	0147	18		0·061 06		39·988
	12	0295	31		29		976
	13	0442	44		51		964
	24	0590	57		74		952
	25	0737	70		97		940
	36	0885	83		0·062 20		928
2	1	1032	96		42		915
	12	1180	0·184 09		64		903
	13	1327	22		87		891
	24	1475	35		0·063 09		879
	25	1622	48		32		867
	36	1769	61		54		39·855 / 40·146
3	1	1917	74		77		40·134
	12	2064	87		0·064 00		122
	13	2212	99		23		110
	24	2359	0·185 12		45		097
	25	2507	25		68		085
	36	2654	38		90		073
4	1	2802	51		0·065 13		061
	12	2949	64		35		048
	13	3097	78		58		036
	24	3244	90	0·000 02	81	0·000 03	024
	25	3391	0·186 03		0·066 04		012
	36	3538	16		26		40·000
5	1	3685	29		49		39·988
	12	3833	42		71		976
	13	3980	55		94		964
	24	4128	69		0·067 16		951
	25	4275	82		39		939
	36	4422	94		61		927
6	1	4569	0·187 07		84		915
	12	4717	21		0·068 07		902
	13	4864	34		29		890
	24	5012	47		52		878
	25	5159	59		74		866
	36	5307	73		97		39·854 / 40·148
7	1	5454	86		0·069 20		40·136
	12	5602	99		42		124
	13	5749	0·188 12		65		111
	24	5897	26		88		099
	25	6044	38		0·070 11		087
	36	6191	51		33		074
8	1	6338	64		56		062
	12	6486	78		78		050
	13	6633	91		0·071 01		037
	24	6781	0·189 04		24		025
	25	6928	18		46		013
	36	7076	31		69		40·000

rth Boundary of each Boundaries.

Difference for 10 Chains.	Quarter Section.
	40·000
	39·988
	976
	964
	952
	940
	928
	915
	903
	891
	879
	867
	39·855
	40·146
	40·134
	122
	110
	097
	085
	073
	061
	048
	036
	024
	012
	40·000
	39·988
	976
	964
	951
	939
	927
	915
	902
	890
	878
	866
	39·854
	40·148
	40·136
	124
	111
	099
	087
	074
	062
	050
	037
	025
	013
	40·000

0·000 03

TABLE IX—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent, &c.—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
9	1	49°·7223	0·189 44		0·071 91		39·988
	12	7371	57		0·072 14		976
	13	7518	70		37		963
	24	7666	83		60		951
	25	7813	96		82		939
	36	7960	0·190 09		0·073 05		926
10	1	8107	23		27		914
	12	8255	36		50		902
	13	8402	49		72		889
	24	8550	62		95		877
	25	8697	76		0·074 19		865
	36	8845	89		41		39·852 / 40·150
11	1	8992	0·191 02		64		40·138
	12	9140	16		86		125
	13	9287	29		0·075 09		113
	24	9435	42		32		100
	25	9582	55		54		088
	36	9729	69		77		075
12	1	49·9876	82		99		40·063
	12	50·0024	95		0·076 23		050
	13	0171	0·192 08		45		038
	24	0319	22		68		025
	25	0466	35		91		013
	36	0614	49		0·077 13		40·000
13	1	0761	62	0·000 02	36	0·000 03	39·988
	12	0908	76		58		975
	13	1056	89		81		963
	23	1203	0·193 02		0·078 03		950
	25	1351	16		27		938
	36	1498	29		50		925
14	1	1645	42		72		913
	12	1793	55		95		900
	13	1940	69		0·079 17		888
	24	2087	83		40		875
	25	2235	96		63		863
	36	2382	0·194 09		85		39·850 / 40·152
15	1	2530	23		0·080 08		40·139
	12	2677	36		31		127
	13	2824	49		54		114
	24	2972	63		77		101
	25	3119	77		99		089
	36	3266	90		0·081 22		076
16	1	3414	0·195 03		45		063
	12	3561	17		67		051
	13	3709	31		90		038
	24	3856	44		0·082 13		025
	25	4003	57		36		013
	36	4151	71		59		40·000
17	1	4298	85		81		39·987
	12	4446	98		0·083 04		975
	13	4593	0·196 11		27		962
	24	4741	25		50		949
	25	4888	39		72		937
	36	5035	52		95		924

TABLE IX—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent, &c.—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
18	1	50°·5182	0·196 66		0·084 17		39·911
	12	5330	80		40		899
	13	5477	93		63		886
	24	5625	0·197 06		86		873
	25	5772	20		0·085 09		861
	36	5920	34		32		39·848 40·153
19	1	6067	47		54		40·140
	12	6714	61		77		128
	13	6362	75		0·086 00		115
	24	6509	88		22		102
	25	6656	0·198 02		45		089
	36	6804	15		68		077
20	1	6951	29		91		064
	12	7098	43		0·087 14		051
	13	7246	56		37		038
	24	7393	70		60		026
	25	7540	84		82		013
	36	7688	97		0·088 05		40·000
21	1	7835	0·199 11		28		39·987
	12	7983	25		50		974
	13	8130	39		73		961
	24	8278	52		96		949
	25	8425	65		0·089 19		936
	36	8572	79		42		923
22	1	8719	93	0·000 02	65	0·000 03	910
	12	8867	0·200 07		88		898
	13	9014	21		0·090 10		885
	24	9162	35		33		872
	25	9309	48		56		859
	36	9457	62		79		39·846 40·155
23	1	9604	75		0·091 02		40·142
	12	9751	89		25		129
	13	9899	0·201 03		48		116
	24	51·0046	17		70		103
	25	0193	31		93		090
	36	0341	45		0·092 16		073
24	1	0488	59		39		065
	12	0635	72		62		052
	13	0783	86		84		039
	24	0930	0·202 00		0·093 07		026
	25	1077	14		30		013
	36	1225	28		53		40·000
25	1	1372	42		76		39·987
	12	1520	56		99		974
	13	1667	69		0·094 22		961
	24	1815	83		44		948
	25	1962	97		67		935
	36	2109	0·203 11		90		922
26	1	2256	25		0·095 13		909
	12	2404	39		36		896
	13	2551	53		59		883
	24	2699	67		82		870
	25	2846	81		0·096 04		857
	36	2994	95		28		39·844 40·157

TABLE IX—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent, &c.—*Continued.*

Township.	Section.	Latitude Φ	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
27	1	51°·3141	0·204 09		0·096 51		40·144
	12	3288	23		73		131
	13	3436	36		96		118
	24	3583	50		0·097 19		105
	25	3730	64		42		092
	36	3878	78		65		078
28	1	4025	92		88		065
	12	4172	0·205 06	0·000 02	0·098 11	0·000 03	052
	13	4320	20		34		039
	24	4467	34		57		026
	25	4614	48		79		013
	36	4762	62		0·099 02		40·000
29	1	4909	76		25		39·987
	12	5056	90		48		974
	13	5204	0·206 04		71		961
	24	5351	19		94		947
	25	5498	33		0·100 17		934
	36	5646	47		40		921
30	1	5793	61		63		908
	12	5940	75		86		894
	13	6088	89		0·101 09		881
	24	6235	0·207 03		32		868
	25	6382	17		54		855
	36	51·6530	0·207 31		0·101 78		39·842
41	36	52·6255	0·216 79		0·116 99		39·918
42	1	6402	94		0·117 22		904
	12	6549	0·217 09		45		891
	13	6697	24		69		877
	24	6844	38		92		863
	25	6991	53		0·118 15		850
	36	7139	68		38		39·836 / 40·166
43	1	7286	82		61		152
	12	7433	96		84		138
	13	7581	0·218 11		0·119 08		124
	24	7728	26		30		111
	25	7875	40		54		097
	36	8023	55		77		083
44	1	8170	70		0·120 00		069
	12	8317	85		24		056
	13	8465	0·219 00		46		042
	24	8612	14		70		028
	25	8759	29		93		014
	36	8907	44		0·121 16		40·000
45	1	9054	58		40		39·986
	12	9201	73	0·000 02	62	0·000 03	972
	13	9349	88		86		958
	24	9496	0·220 03		0·122 09		945
	25	9643	18		32		931
	36	9791	33		56		917
46	1	9938	48		79		903
	12	53·0085	63		0·123 02		890
	13	0233	77		25		876
	24	0380	92		49		862

TABLE IX—*Concluded.*

LATITUDE, with Logarithms of Secant and Tangent, &c.—*Concluded.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	25	53°·0527	0·221 07		0·123 71		848
	36	0675	21		95		39·834
							40·168
47	1	0822	36				
	12	0969	51		0·124·19		40·154
	13	1117	66		41		140
	24	1264	81		65		126
	25	1411	96		88		112
	36	1559	0·222 11		0·125·12		098
					34		084
48	1	1706	26		58		070
	12	1853	41		81		056
	13	2001	56		0·126 04		042
	24	2148	71		28		028
	25	2295	86		51		014
	36	2443	0·223 00		74		40·000

TABLE X.

LATITUDE, with Logarithms of Secant and Tangent for the north boundary of each Section, and width of Quarter Sections on such boundaries.

(Third System of Survey.)

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	36	49°·0000	0·183 06		0·060 84		40·000
1	1	0147	19		0·061 06		39·988
	12	0291	31		28		976
	13	0438	44		51		964
	24	0582	57		73		953
	25	0729	69		95		941
	36	0874	82		0·062 17		929
2	1	1020	95	0·000 02	40	0·000 03	917
	12	1165	0·184 08		62		905
	13	1311	20		85		893
	24	1456	33		0·063 07		882
	25	1603	46		29		870
	36	1747	59		51		39·858
							40·143
3	1	1894	71		74		131
	12	2039	84		96		119
	13	2185	97		0·064 18		107
	24	2330	0·185 10		41		095
	25	2476	23		63		084
	36	2621	35		85		072
4	1	2768	48		0·065 08		060
	12	2912	61		30		048

&c.—*Concluded.*

Difference for 10 Chains.	Quarter Section.
	848
	39·834
	40·168
	40·154
	140
	126
	112
	098
	084
	070
	056
	042
	028
	014
	40·000

:th ndary of each
ind es.

Difference for 10 Chains.	Quarter Section.
	40·000
	39·988
	976
	964
	953
	941
	929
0·000 03	917
	905
	893
	882
	870
	39·858
	40·143
	131
	119
	107
	095
	084
	072
	060
	048

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	13	49°·3059	0·185 74		0·065 52		40·036
	24	3203	87		74		024
	25	3350	0·186 00		97		012
	36	3495	12		0·066 19		000
5	1	3641	25		42		39·988
	12	3786	38		64		976
	13	3932	51		86		964
	24	4077	64		0·067 08		952
	25	4224	77		31		940
	36	4368	90		53		928
6	1	4515	0·187 03		76		916
	12	4659	15		98		904
	13	4806	28		0·068 20		892
	24	4951	41		43		880
	25	5097	54		65		868
	36	5242	67		87		39·858
							40·145
7	1	5388	80		0·069 10		133
	12	5533	93		32		121
	13	5680	0·188 06		54		109
	24	5824	19		77		097
	25	5971	32		99		085
	36	6115	45		0·070 21		073
8	1	6262	58	0·000 02	44	0·000 03	060
	12	6407	71		66		048
	13	6553	84		89		036
	24	6698	97		0·071 11		024
	25	6844	0·189 10		33		012
	36	6989	23		56		000
9	1	7136	36		78		39·988
	12	7280	49		0·072 00		976
	13	7427	62		23		964
	24	7571	75		45		951
	25	7718	88		68		939
	36	7863	0·190 01		90		927
10	1	8009	14		0·073 12		915
	12	8154	27		35		903
	13	8300	40		57		891
	24	8445	53		79		879
	25	8592	66		0·074 02		867
	36	8736	79		24		39·855
							40·147
11	1	8883	93		47		135
	12	9027	0·191 06		69		122
	13	9174	19		92		110
	24	9319	32		0·075 14		098
	25	9465	45		36		086
	36	9610	58		59		073
12	1	9756	71		81		061
	12	9901	84		0·076 03		050
	13	50°·0047	88		26		037
	24	0192	0·192 11		48		024

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	25	50°·0339	0·192 24		0·076 71		40·012
	36	0483	37		93		000
13	1	0630	50		0·077 16		39·988
	12	0775	63		38		975
	13	0921	77		60		963
	24	1066	90		83		951
	25	1212	0·193 03		0·078 05		939
	36	1357	16		28		926
14	1	1503	29		50		914
	12	1648	43		72		902
	13	1795	56		95		890
	24	1939	69		0·079 17		877
	25	2086	82		40		865
	36	2230	96		62		39·853 40·149
15	1	2377	0·194 09		85		137
	12	2522	22		0·080 07		124
	13	2668	35		30		112
	24	2813	49		52		099
	25	2959	62		75		087
	36	3104	75		97		074
16	1	3250	89		0·081 20		062
	12	3395	0.195 02		42		050
	13	3542	15		64		037
	24	3686	28		87		025
	25	3833	42		0·082 09		012
	36	3977	55	0·000 02	32	0·000 03	000
17	1	4124	69		54		39·988
	12	4269	82		77		975
	13	4415	95		99		963
	24	4560	0·196 09		0·083 22		950
	25	4706	22		44		940
	36	4851	35		67		925
18	1	4997	49		89		913
	12	5142	62		0·084 12		901
	13	5289	76		34		888
	24	5433	89		56		876
	25	5580	0·197 02		79		863
	36	5724	16		0·085 01		39·851 40·150
19	1	5871	29		24		138
	12	6016	43		46		125
	13	6162	56		69		113
	24	6307	69		91		100
	25	6453	83		0·086 14		088
	36	6598	96		36		075
20	1	6744	0·198 10		59		063
	12	6889	23		81		050
	13	7035	37		0·087 04		038
	24	7180	50		27		025
	25	7327	64		49		013
	36	7471	77		72		000
21	1	7618	91		94		39·987
	12	7762	0·199 04		0·088 17		975

Section, and width of

Difference for 10 Chains.	Quarter Section.
	40·012
	000
	39·988
	975
	963
	951
	939
	926
	914
	902
	890
	877
	865
	39·853 / 40·149
	137
	124
	112
	099
	087
	074
	062
	050
	037
	025
	012
	000
	39·988
	975
	963
	950
	940
	925
	913
	901
	888
	876
	863
	39·851 / 40·150
	138
	125
	113
	100
	088
	075
	063
	050
	038
	025
	013
	000
	39·987
	975

0·000 03

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter-Section.
	13	50°·7909	0·199 18		0·088 39		39·962
	24	8054	31		62		950
	25	8200	45		84		937
	36	8345	58		0·089 07		925
22	1	8491	72		29		912
	12	8636	85		52		899
	13	8782	99		74		887
	24	8927	0·200 13	0·000 02	97	0·000 03	874
	25	9073	26		0·090 20		862
	36	9218	40		42		39·849 / 40·152
23	1	9365	53		65		140
	12	9509	67		87		127
	13	9656	81		0·091 10		114
	24	9800	94		32		102
	25	9747	0·201 08		55		089
	36	51°·0091	21		77		076
24	1	0238	35		0·092 00		064
	12	0383	49		22		051
	13	0529	63		45		038
	24	0674	76		68		025
	25	0820	90		90		013
	36	0965	0·202 03		0·093 13		000
25	1	1111	17		35		39·987
	12	1256	31		58		975
	13	1402	44		81		962
	24	1547	58		0·094 03		949
	25	1694	72		26		936
	36	1838	58		48		924
26	1	1985	99		71		911
	12	2129	0·203 13		93		898
	13	2276	27		0·095 16		885
	24	2420	40		39		873
	25	2567	54		61		860
	36	2712	68		84		39·847 / 40·154
27	1	2858	82		0·096 07		141
	12	3003	95		29		129
	13	3149	0·204 09	0·000 02	52	0·000 03	116
	24	3294	23		74		103
	25	3440	37		97		090
	36	3585	51		0·097 19		077
28	1	3731	64		42		064
	12	3876	78		65		051
	13	4023	92		87		039
	24	4167	0·205 06		0·098 10		026
	25	4314	20		33		013
	36	4458	33		55		000
29	1	4605	47		78		39·987
	12	4749	61		0·099 00		974
	13	4896	75		29		962
	24	5040	89		46		949
	25	5187	0·206 03		69		936
	36	5332	17		91		923

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
30	1	51°·5478	0·206 31		0·100 14		
	12	5263	44		36		910
	13	5769	58		59		897
	24	5914	72		82		884
	25	6060	86		0·101 05		39·871
	36	6205	0·207 00		27		858
							39·846
							40·156
31	1	6351	14		50		
	12	6496	28		72		143
	13	6642	42		95		130
	24	6787	56		0·102 18		117
	25	6934	70		41		104
	36	7078	84		63		091
							078
32	1	7225	99		86		
	12	7369	0.208 12		0·103 08		065
	13	7516	26		31		052
	24	7660	40		54		039
	25	7807	54		77		026
	36	7951	68		99		013
							000
33	1	8098	82		0·104 22		
	12	8243	96		45		39·987
	13	8389	0·209 10		68		974
	24	8534	24		90		961
	25	8680	38		0·105 13		948
	36	8825	52		35		935
							922
34	1	8971	66	0·000 02	58	0·000 03	
	12	9116	80		81		909
	13	9262	94		0·106 04		896
	24	9407	0·210 08		26		883
	25	9553	22		49		869
	36	9698	36		72		856
							39·843
							40·158
35	1	9844	51		95		
	12	9989	65		0·107 17		145
	13	52°·0135	79		40		132
	24	0280	93		63		119
	25	0427	0·211 07		86		106
	36	0571	21		0·108 08		092
							079
36	1	0718	36		31		
	12	0862	50		54		066
	13	1009	64		77		053
	24	1153	78		99		040
	25	1300	92		0·109 22		026
	36	1444	0·212 06		45		013
							000
37	1	1591	21		68		
	12	1735	35		90		39·987
	13	1882	49		0·110 13		974
	24	2027	63		36		960
	25	2173	77		59		947
	36	2318	92		81		934
							921
38	1	2464	0·213 06		0·111 04		
	12	2609	20		27		907
	13	2755	34		50		894
	24	2900	49		73		881
							868

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	25	52°·3046	0·213 63		0·211 96		855
	36	3191	77		0·112 18		39·841 / 40·160
39	1	3337	92	0·000 02	41	0·000 03	147
	12	3482	0·214 06		64		134
	13	3628	20		87		120
	24	3773	34		0·113 09		107
	25	3919	49		32		093
	36	4064	63		55		080
40	1	4210	77		78		067
	12	4355	92		0·114 01		053
	13	4501	0·215 06		24		040
	24	4646	20		46		027
	25	4794	35		69		013
	36	4937	49		92		000
41	1	5084	64		0·115 15		39·987
	12	5228	78		38		973
	13	5375	92		61		960
	24	5519	0·216 07		83		946
	25	5666	21		0·116 06		933
	36	5810	35		29		920
42	1	5957	50		52		906
	12	6101	64		75		893
	13	6248	79		98		879
	24	6392	93		0·117 21		866
	25	6539	0·217 08		44		853
	36	6683	22		66		39·839 / 40·162
43	1	6830	37		89		149
	12	6974	51		0·118 12		135
	13	7121	66		35		122
	24	7266	80		58		108
	25	7412	95		81		095
	36	7557	0·218 09		0·119 04		081
44	1	7703	24		27		068
	12	7848	38		49		054
	13	7994	53		73		041
	24	8139	67		95		027
	25	8285	82		0·120 18		014
	36	8430	96		41		000
45	1	8576	0·219 11	0·000 02	64	0·000 03	39·986
	12	8721	25		87		973
	13	8867	40		0·121 10		950
	24	9012	55		33		946
	25	9158	69		56		932
	36	9303	84		79		919
46	1	9449	98		0·122 02		905
	12	9594	0·220 13		25		891
	13	9740	28		48		878
	24	9885	42		70		864
	25	53°·0031	57		93		851
	36	0176	71		0·123 16		39·837 / 40·164

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
47	1	53°·0321	0·220 86		0·123 39		
	12	0467	0·221 01				40·151
	13	0612	15		62		137
	24	0758	30		85		123
	25	0903	45		0·124 08		110
	36	1049	59		31		096
48	1	1195	74		54		082
	12	1340	89		77		068
	13	1486	0·222 04		0·125 00		055
	24	1631	18		23		041
	25	1777	33		46		027
	36	1922	48		69		014
49	1	2068	63		92		000
	12	2213	77		0·126 15		
	13	2359	92		38		39·986
	24	2504	0·223 07		61		972
	25	2650	22		84		958
	36	2795	36		0·127 07		945
50	1	2941	51		30		931
	12	3086	66		53		917
	13	3233	81		76		903
	24	3377	96		99		889
	25	3524	0·224 10		0·128 22		875
	36	3668	25		45		861
51	1	3815	40	0·000 02	68	0.000 03	848
	12	3959	55		91		{39·834 40·166
	13	4106	70		0·129 14		153
	24	4250	85		37		139
	25	4397	0·225 00		60		125
	36	4541	14		83		111
52	1	4688	29		0·130 06		097
	12	4832	44		30		083
	13	4979	59		53		069
	24	5123	74		76		055
	25	5270	89		99		042
	36	5414	0·226 04		0·131 23		028
53	1	5561	19		45		014
	12	5705	34		68		000
	13	5852	49		91		39·986
	24	5996	63		0·132 14		972
	25	6143	79		37		958
	36	6287	93		60		944
54	1	6434	0·227 08		83		930
	12	6578	23		0·133 07		917
	13	6725	38		30		903
	24	6869	53		53		890
	25	7016	68		76		875
	36	7160	83		99		861
55	1	7307	99		0·134 22		847
	12	7451	0·228 13		45		{39·834 40·169
	13	7598	29		68		155
	24	7742	44		91		140
					0·135 14		126
							112

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	25	53°·7889	0·228 59		0·135 38		40·098
	36	8033	74		61		084
56	1	8180	89		84		070
	12	8324	0·229 04		0·136 07		056
	13	8471	19	0·000 02	30	0·000 03	042
	24	8615	34		53		028
	25	8762	49		77		014
	36	8906	64		0·137 00		000
57	1	9052	79		23		39·986
	12	9197	95		46		972
	13	9343	0·230 10		69		958
	24	9488	25		92		944
	25	9634	40		0·138 16		930
	36	9779	55		39		915
58	1	9925	70		62		901
	12	54°·0070	85		85		887
	13	0216	0·231 01		0·139 08		873
	24	0361	16		31		859
	25	0507	31		55		845
	36	0652	46		78		39·831 40·171
59							
	1	0798	62		0·140 01		157
	12	0943	77		24		142
	13	1089	92		48		128
	24	1234	0·232 07		71		114
	25	1380	23		94		100
	36	1525	38		0·141 17		085
60	1	1671	53		41		071
	12	1816	68		64		057
	13	1962	84		87		043
	24	2107	99		0·142 10		028
	25	2253	0·233 14		34		014
	36	2398	29		57		000
61	1	2544	45		80		39·986
	12	2689	60		0·143 03		971
	13	2835	76		27		957
	24	2980	91		50		943
	25	3126	0·234 06		73		929
	36	3271	21		96		914
62	1	3417	37		0·144 20		900
	12	3562	52		43		886
	13	3708	68		66		872
	24	3853	83		89		857
	25	3999	98		0·145 13		843
	36	4144	0·235 14		36		39·829 40 173
63	1	4290	29		59		159
	12	4435	45		83		144
	13	4581	60	0·000 02	0·146 06	0·000 03	130
	24	4725	75		29		115
	25	4872	91		53		101
	36	5016	0·236 06		76		086
64	1	5163	22		99		072
	12	5307	37		0·147 22		058

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	13	54°·5454	0·236 53		0·147 46		40·043
	24	5598	68		69		029
	25	5745	84		93		014
	36	5889	99		0·148 16		000
65	1	6036	0·237 15		39		986
	12	6180	30		63		971
	13	6327	46		86		957
	24	6471	61		0·149 09		942
	25	6618	77		33		928
	36	6762	92		56		913
66	1	6909	0·238 08		80		899
	12	7053	24		0·150 03		884
	13	7199	39		26		870
	24	7344	55		50		855
	25	7490	70		73		841
	36	7635	86		96		{ 39·827 40·175
67	1	7781	0·239 02		0·151 20		161
	12	7926	17		43		146
	13	8072	33		67		131
	24	8217	49		90		117
	25	8363	64		0·152 13		102
	36	8508	80		37		088
68	1	8654	96		60		073
	12	8799	0·240 11	0·000 02	84	0·000 03	058
	13	8945	27		0·153 07		044
	24	9090	43		31		029
	25	9236	58		54		015
	36	9381	74		77		000
69	1	9527	90		0·154 01		39·985
	12	9672	0·241 05		24		971
	13	9818	21		48		956
	24	9962	37		71		941
	25	55°·0109	53		95		927
	36	0253	68		0·155 18		912
70	1	0400	84		42		898
	12	0544	0·242 00		65		883
	13	0691	16		89		868
	24	0835	31		0·156 12		854
	25	0982	47		36		839
	36	1126	63		59		{ 39·824 40·177
71	1	1274	79		83		163
	12	1417	95		0·157 06		148
	13	1563	0·243 11		30		133
	24	1708	26		53		118
	25	1854	42		77		104
	36	1999	58		0·158 00		089
72	1	2145	74		24		40·074
	12	2290	90		47		059
	13	2436	0·244 06		71		044
	24	2581	22		94		030
	25	2727	38		0·159 18		015
	36	2872	53		41		000

TABLE X—*Continued.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Continued.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
73	1	55°·3018	0·244 69		0·159 65		39·985
	12	3163	85		89		970
	13	3309	0·245 01		0·160 12		956
	24	3454	17		36		941
	25	3600	33		59		926
	36	3744	49		83		911
74	1	3891	65		0·161 07		896
	12	4035	81		30		881
	13	4182	97		54		867
	24	4326	0·246 13		77		852
	25	4473	29		0·162 01		837
	36	4617	45		24		39·822 40·180
75	1	4764	61		48		165
	12	4908	77		72		150
	13	5054	93		95		135
	24	5199	0·247 09		0·163 19		120
	25	5345	25		43		105
	36	5490	41		66		090
76	1	5636	57		90		075
	12	5781	73		0·164 13		060
	13	5927	90		37		045
	24	6072	0.248 06		61		030
	25	6218	22		85		015
	36	6363	38		0·165 08		000
77	1	6509	54	0·000 02	32	0·000 03	39·985
	12	6654	70		55		970
	13	6800	86		79		955
	24	6944	0·249 02		0·166 03		940
	25	7091	19		27		925
	36	7235	35		50		910
78	1	7382	51		74		895
	12	7526	67		98		880
	13	7672	83		0·167 21		865
	24	7817	0·250 00		45		850
	25	7963	16		69		835
	36	8108	32		92		39·820 40·182
79	1	8254	48		0·168 16		167
	12	8399	64		40		152
	13	8545	81		64		137
	24	8690	97		87		122
	25	8836	0·251 13		0·169 11		106
	36	8981	30		35		091
80	1	9127	46		59		076
	12	9272	62		82		061
	13	9418	79		0·170 06		046
	24	9562	95		30		030
	25	9709	0·252 11		54		015
	36	9853	27		77		000
81	1	56°·0000	44		0·171 01		39·985
	12	0144	60		25		970
	13	0291	77		49		954
	24	0435	93		72		939

TABLE X—*Concluded.*

LATITUDE, with Logarithms of Secant and Tangent for each Section, and width of Quarter Sections—*Concluded.*

Township.	Section.	Latitude Φ.	Sec Φ.	Difference for 10 Chains.	Tan Φ.	Difference for 10 Chains.	Quarter Section.
	25	56°·0581	0·253 09		0·171 96		39·924
	36	0726	26		0·172 20		909
82	1	0872	42		44		893
	12	1017	58		68		878
	13	1163	75		92		863
	24	1308	91		0·173 15		848
	25	1454	0·254 08		39		833
	36	1599	24		63		39·817 40·185

TABLE XI.

To Convert Chains into Decimals of a Township Side.

Chains.	Equivalent Decimal of a Township Side.			Chains.	Equivalent Decimal of a Township Side.		
	Side = 489c.	Side = 486c.	Side = 483c.		Side = 489c.	Side = 486c.	Side = 483c.
1	0·00204	0·00206	0·00207	30	0·06135	0·06173	0·06211
2	·00409	·00412	·00414	40	·08180	·08230	·08282
3	·00613	·00617	·00621	50	·10225	·10288	·10352
4	·00818	·00823	·00828	60	·12270	·12346	·12422
5	·01022	·01029	·01035	70	·14315	·14403	·14493
6	·01227	·01235	·01242	80	·16360	·16461	·16563
7	·01431	·01440	·01449	90	·18405	·18519	·18634
8	·01636	·01646	·01656	100	·20450	·20576	·20704
9	·01840	·01852	·01863	200	·40900	·41152	·41408
10	·02045	·02058	·02070	300	·61350	·61728	·62112
20	·04090	·04115	·04141	400	·81800	·82305	·82816

Section, and width of

Difference for 10 Chains.	Quarter Section.
	39·924
	909
	893
	878
	863
	848
	833
	39·817
	40·185

Side.

nal of a Township Side.

e = 486c.	Side = 483c.
0·06173	0·06211
·08230	·08282
·10288	·10352
·12346	·12422
·14403	·14493
·16461	·16563
·18519	·18634
·20576	·20704
·41152	·41408
·61728	·62112
·82305	·82816

TABLE XII.

CORRECTIONS to be applied to the tabular quantities in Table No. VII when the north side of the road allowance on Correction Lines is run instead of the south; also correction to road allowance on account of curvature.

Number of Correction Line.	Correction to Chord Azimuth	Correction to Deflection Offset (for one chain distance).	Correction to width of road allowance on account of curvature.									
			jog = 30 chs.	jog = 40 chs.	jog = 50 chs.	jog = 60 chs.	jog = 70 chs.	jog = 80 chs.	jog = 90 chs.	jog = 100 chs.	jog = 110 chs.	jog = 120 chs.
	"	inches.	lks.	lks.	lks.	lks.	lks.	lks.	lks.	lks.	lks.	lks.
1st	−1·3	+0·010	2·5	3·2	3·9	4·6	5·2	5·8	6·4	7·0	7·5	7·9
11th	−1·7	+0·013	2·8	3·7	4·5	5 2	6·0	6·7	7·3	7·9	8·5	8·9
21st	−2·2	+0·017	3·2	4·2	5·2	6·0	6·9	7·7	8·4	9·1	9·8	10·4
31st	−2·9	+0·022	3·7	4·8	5·9	6·9	7·9	8·8	9·6	10·4	11·2	11·9

TABLE XIII.

SHOWING the difference of Latitude between Township Corners and Section and Quarter Section Posts on a Township Chord.

Number of Line.	$d\Phi$ For ½ sec. from Corner.	$d\Phi$ For 1 sec. from Corner.	$d\Phi$ For 1½ sec. from Corner.	$d\Phi$ For 2 secs. from Corner.	$d\Phi$ For 2½ secs. from Corner.	$d\Phi$ For 3 secs. from Corner.
	"	"	"	"	"	"
1st Base	0·02 lks.	0·04 lks.	0·05 lks.	0·06 lks.	0·07 lks.	0·07 lks.
do	3·2 "	5·9 "	8·0 "	9·5 "	10·3 "	10·8 "
11th Base	0·02 lks.	0·04 lks.	0·06 lks.	0·07 lks.	0·08 lks.	0·08 lks.
do	3·6 "	6·7 "	9·1 "	10·8 "	11·8 "	12·1 "
21st Base	0·03 lks.	0·05 lks.	0·07 lks.	0·08 lks.	0·09 lks.	0·09 lks.
do	4·2 "	7·7 "	10·3 "	12·3 "	13·3 "	13·8 "
31st Base	0·03 lks.	0·06 lks.	0·08 lks.	0·09 lks.	0·10 lks.	0·11 lks.
do	4·8	8·8	12·0	14·4	15·6	16·2